Strategy Choices in Addition and Subtraction: How Do Children Know What to Do?

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Out of all of the problem-solving strategies that people could use, how do they decide which ones to use? Even a task as mundane as spelling reveals the variety of strategies that people can employ. Suppose that someone was trying to spell the word “accommodation.” One approach would be to retrieve the spelling. An alternative would be to try to form an image of what the word looks like. Another possibility would be to write out several alternative spellings and to try to recognize the correct one. Yet another strategy would be to look up the word in a dictionary. These variations in strategies are not only “between-subjects” phenomena. Individuals often use each of the approaches at different times, even on a single word.

Good reasons exist for people to know and to use multiple strategies for achieving a goal. Strategies differ in their accuracy, in the amounts of time they require, in their memory demands, and in the range of problems to which they apply. Strategy choices involve tradeoffs among these properties so that people can cope with cognitive and situational constraints. These cognitive and situational constraints can vary from moment to moment, even within what ordinarily is viewed as a single problem. The broader the range of strategies that people know, the more precisely they can shape their approaches to meet these changing circumstances. As becomes evident in the course of this chapter, even young children can choose strategies in adaptive ways. Our goals are to specify how they make such strategy choices, how the ability to make them develops, and what functions the strategy-choice process serves.

The particular strategy choices that we examine are those that preschoolers make in solving simple arithmetic problems. Previous reports indicate that children use a variety of strategies to add and subtract. They count from 1, count on
from the first or the higher number, put up their fingers and count them, tap their feet rhythmically, and decompose complex problems into simpler ones (e.g., \(3 + 4 = (3 + 3) + 1\)). The reports of these visible and audible strategies have been largely anecdotal, but they have been sufficiently persistent to leave little doubt of their existence. Left undescribed, however, have been the mechanisms by which children arrive at a strategy on a particular problem and whether their use of overt strategies results in more effective problem solving. These issues are the focus of the present research.

We begin the chapter by discussing current views on how people select strategies and on how they solve simple addition problems. Then we examine several recent experiments indicating that a single child may use as many as four distinct strategies in solving such problems. The particular strategy that children use on each problem turns out to be closely related to the problem’s difficulty. Next we present a distribution of associations model that accounts for this close relation between strategy use and problem difficulty as well as for the existence of the four strategies and for their temporal characteristics. Following this, we present evidence that the model applies to performance on a broader range of addition problems and also to performance on subtraction problems. Then we present a second distribution of associations model that subsumes the first model’s assumptions about performance but that also accounts for how strategy choices on simple addition and subtraction problems develop. This second model is expressed as a running computer simulation that initially generates poor performance but that learns from its experience in such a way that it eventually produces patterns of strategy use, solution times, and correct answers and errors much like those of 4- and 5-year-olds. Finally, we speculate about how the revised model might apply to other domains, such as spelling and beginning reading, and discuss its general implications for how people arrive at strategies.

A BRIEF REVIEW OF THE LITERATURES ON STRATEGY CHOICE AND ELEMENTARY ADDITION

The Issue of Strategy Choice

Cognitive psychologists who study adults have devoted considerable effort to determining the strategy people use to perform particular tasks. Out of these efforts have come models of sentence verification (Clark & Chase, 1972; Just & Carpenter, 1975; Trabasso, Rollins, & Shaughnessy, 1971), spatial information processing (Cooper & Shepard, 1973; Shepard & Metzler, 1971), transitive inference (H. H. Clark, 1969; Huttenlocher & Higgins, 1971; Sternberg, 1977), and many other tasks. Recently, however, a number of investigators have noted that people’s strategies vary. They have suggested that cognitive theories should explain how people choose among alternative approaches. Consider three recent comments:

Discussion of the appropriate models for psycholinguistic tasks is usually couched in general terms (i.e., “What models apply to people?”). Our results can be seen as a reminder that this approach is too simplistic. The same ostensibly linguistic task can be approached in radically different ways by different people (MacLeod, Hunt, & Mathews, 1978, p. 506).

Too often psychologists set out to study the way that a task is performed, and miss one of the most interesting and general aspects of human cognitive performance: that there is more than one way to skin a cat. Once we accept this flexibility as a significant characteristic of the way that humans think and learn, rather than a troublesome source of variation in our data, it becomes important to understand the factors that control the adoption of one strategy over others (Farah & Kosslyn, 1982, p. 164).

Information processing psychologists have little to say on “how it is that the child knows what to do” and “what” inside the child’s head makes the decisions (Gardner, 1982, p. 421).

These opinions and recommendations have been accompanied by a growing amount of research demonstrating that people do use diverse strategies on tasks for which cognitive psychologists previously had proposed a single model. Hunt and his colleagues demonstrated that different people use different approaches to verify sentences (MacLeod et al., 1978; Mathews, Hunt, & MacLeod, 1980). Cooper and her colleagues described alternative strategies that people use to perform a spatial comparison task (Cooper & Regan, 1982; Glushko & Cooper, 1978). Egan and Grimes-Farrow (1982) and Sternberg and Weil (1980) identified several strategies that people use to draw transitive inferences.

All of these studies of strategy differences followed the comparative approach. That is, they defined groups on some preexisting status variable and demonstrated that group membership predicted behavioral differences. Hunt’s, Egan’s, Cooper’s, and Sternberg’s studies all used spatial and verbal abilities as the covariate. People relatively high in spatial ability used one strategy; people relatively high in verbal ability used another. Cross-cultural investigators (e.g., Wagner, 1978) and those interested in aging (e.g., Reder, 1982) have also contrasted the strategies used by members of different groups.

Developmental psychologists have a longer tradition of attending to strategy differences than do psychologists who study adults. One of the central phenomena of developmental psychology is that people of different ages often vary in their approaches. Five-year-olds rarely rehearse when asked to remember
arbitrary lists of words; 8-year-olds usually do rehearse; 11-year-olds rehearse in a more comprehensive, less repetitive, way than 8-year-olds (Flavell, Beach, & Chinsky, 1966; Naus, Ornstein, & Aivano, 1977). Five-year-olds judge which side of a balance scale will go down solely on the basis of weight; 9-year-olds usually consider both weight and distance from the fulcrum but do not know the proportionality rule for combining them; at least some 18-year-olds compute relative torques on the two sides of the balance (Inhelder & Piaget, 1958; Siegler, 1976). Similar strategy differences among different-aged children have been found on analogical reasoning, probability learning, visual scanning, and many other tasks (Sternberg & Rifkin, 1979; Vrupilol, 1968; Weir, 1964).

Investigations showing that people of a particular age, ability profile, or culture tend to use a particular strategy are useful in at least three ways. They document the range of strategies that people can use to solve the problem. They illustrate that different people spontaneously choose different strategies. They show that these choices frequently correlate with group membership. However, they may also divert attention from an issue of even greater interest: how strategy choices are made. A person high in spatial ability need not always use a spatially oriented strategy, nor one high in verbal ability a verbally oriented one. Egan and Grimes-Farrow (1982), Mathews et al. (1980), and Sternberg and Ketron (1982) have all demonstrated that people who ordinarily use one strategy will use a different one if instructed to do so. Cooper and Regan (1982) showed that aspects of the particular stimulus configuration also influence which strategy people adopt. Even if people were not so flexible, they would still need strategy-choice procedures. Before entering the experimental situation, most people do not have extensive experience with the tasks that they encounter (e.g., solving three-term syllogism problems). How does a person high in spatial aptitude know to use a spatially rather than a verbally oriented strategy?

One set of efforts to explain strategy choices is included under the heading of metacognition. Underlying much metacognitive research is the plausible belief that people use explicit knowledge of their cognitive capacities, available strategies, and task demands to determine which strategy to use. When confronted with a problem, they might reason, "This is a difficult problem, too difficult to solve without a powerful strategy such as x, I'd better use x."

Several difficulties have arisen in metacognitive research that make this mode of explanation less promising than it once appeared. On an empirical level, research has revealed only modest correlations between metacognitive knowledge and performance measures (see reviews by Flavell & Wellman, 1977, and by Cavanaugh & Perlmutter, 1982). On a theoretical level, there is considerable lack of clarity about how metacognitive knowledge would lead to strategy choices. Do people make explicit judgments about their intellectual capacities, about available strategies, and about task demands every time they face a task that they could perform in two or more ways? If not, how do they decide on which tasks to do so? Do they consider every strategy that they conceivably could use on the task or only some subset of them? If only a subset, how do they decide which ones? How do people know what their cognitive capacity will be or what strategies they could apply when they are presented a novel task? Determining how metacognitive knowledge leads to strategy choices is much more complex than initially might be supposed.

There also appears to be a mismatch between the seemingly useful strategies that very young children at times adopt and their apparent lack of metacognitive knowledge. DeLoache (this volume) reported that when 1½-year-olds in a laboratory situation saw an experimenter hide objects that they later needed to find, they engaged in more labeling and other types of discussion of the objects than when the experimenter hid them in the children's own homes. The children did not discuss the objects at all when they remained visible throughout the waiting period. DeLoache concluded that the children used the labeling and discussion strategy when they needed it to keep alive a memory trace that otherwise might fade. This conclusion seemed consistent with the data. But how did the 1½-year-olds make this decision? Did they possess sufficient knowledge of their memory capacities and of task demands to anticipate that their memory traces might fade if they did not talk about the hidden objects?

In sum, a decade of research on people's explicit metacognitive knowledge has not explained how they arrive at their strategies. This lack of success raises an intriguing possibility. Perhaps people can arrive at adaptive strategies without explicitly considering capacity limitations, available strategies, and task demands. This possibility will be the focus of the present chapter.

The development of addition skills. Research on how addition skills develop, like research on strategy choice, has followed the comparative approach of equating the performance of one group with one strategy and the performance of another group with another. The two best-known models are those of Groen and Parkman (1972) and Ashcraft (1982). Groen and Parkman proposed the min model. This model indicates that when people are given a problem with two addends, they add by selecting the larger addend and counting up from it the number of times indicated by the smaller. Groen and Parkman hypothesized that the time needed to identify the larger number was a constant for all problems. Therefore, solution times depended only on the number of increments indicated by the smaller number (hence the name min model). The only exception to this formula involved ties, problems with equal augend and addend. Groen and Parkman suggested that answers to these problems were retrieved directly, at a uniformly rapid rate, so that solution times for all tie problems would be faster than solution times for any other problem (excepting those with zero as an addend, which would be retrieved as quickly as ties, due to their not requiring any increments of the larger number).

Several types of evidence supported this model. Groen and Parkman found that the solution times of both first graders and adults increased by constant
Some Methodological Issues

It is not coincidental that researchers working with phonographic data have tended to postulate a single strategy for all subjects within a given age or ability group, whereas researchers relying on direct observation have postulated multiple strategies. Solution-produced data are behaviorally noisy and sufficiently remote from the processes that produce them only by aggregating over a large number of trials. Even given a large number of trials, it is extraordinarily difficult to infer from a person's solution times that he uses multiple strategies. In contrast, simply behaviorally apparent data can reveal multiple strategies if visible or audible behavior accompanies the strategies. Such observations, however, do not yield behaviorally precise data to allow use of powerful methods for inferring the processes by which children solve the problem.

Video filming young children's problem-solving behavior would help in deciding which strategies are used. The visual record of the strategies used would also allow a more precise measurement of the range of strategies that children used. It would also allow a more precise measurement of the range of strategies that children used. These observations would help in determining how each strategy was executed and how children chose among strategies. Together, these observations would make the strategy choice-traceable.

The possibility that even individual differences in performance can be a variety of strategies at different times and in different conditions. What strategies are used? What are their accuracy and temporal characteristics? How does a person achieve the strategy to use on a given occasion? Does use of different strategies change with age or ability? These questions are addressed in the next section, where we describe a recent empirical study that addressed these issues.

Like the previously cited comparative research, these studies of how people solve simple addition problems highlight the fact that people have different demographic characteristics. Also like the previous studies, they suggest that research into how people solve simple addition problems may be useful in improving educational outcomes.
AN EMPIRICAL STUDY OF CHILDREN’S ADDITION

Method

Siegler and Robinson (1982) examined 4- and 5-year-olds’ addition strategies. The 30 children who participated, 17 boys and 13 girls, were students at a university-run preschool.

Children were videotaped as they solved 25 addition problems. The problems were the possible combinations of augends from one to five and addends from one to five. The instructions were the following:

I want you to imagine that you have a pile of oranges. I’ll give you more oranges to add to your pile; then you need to tell me how many oranges you have altogether.

Okay? You have m oranges, and I’m going to give you n to add to your pile. How many do you have altogether?

After four or five questions, many children indicated that they preferred to hear the problem in the form “How much is m + n?” We complied with their request.

Each child was presented each problem on two occasions; thus, children were eventually presented 50 trials. These 50 trials were divided among six sessions, with eight or nine problems in each session. Children performed the problems while sitting at a desk with a bare top; no external objects were present for them to manipulate while they solved the problems. The experimenter praised the children and gave each child a star following each correct answer. Sessions lasted approximately 5 minutes apiece.

Results

The videotapes revealed four strategies. Three were overt (visible or audible) approaches. Sometimes children raised fingers corresponding to each addend and counted them (the counting-fingers strategy). Other times they lifted fingers corresponding to each addend but answered without counting them (the fingers strategy). Yet other times they counted aloud (or moved their lips in a visible, silent-counting sequence), but their counting did not have any obvious external referent (the counting strategy). The fourth approach involved no visible or audible behavior. For reasons that become apparent in the course of the chapter, we labeled this the retrieval strategy.

As shown in Table 9.1, the four strategies differed in their frequency of use and in their temporal and accuracy characteristics. Of particular interest within the analysis that follows were relative solution times. For each pair of strategies, we compared mean solution times on each of the 25 problems for those trials on which children used one strategy to the mean solution times on trials on that problem on which they used the other. (Only the solution times of children who used both strategies being compared at least twice were included in this analysis). Retrieval was significantly faster than the fingers strategy, t(24) = 3.10, which in turn was faster than the counting-fingers strategy, t(23) = 3.96. Retrieval was also significantly faster than counting, t(23) = 8.87 which in turn was faster than counting fingers, t(23) = 3.91 (all p’s < .01).

The most intriguing finding of the experiment was unexpected. The preschoolers proved to be surprisingly adept at matching their use of strategies with the difficulty of the problems. There was a very close association (r = .91) between percentage of errors on the 25 problems and percentage of use of the three overt strategies on them (Fig. 9.1). Children most frequently used overt strategies on exactly those problems that were the most difficult to answer correctly. Percentage of overt strategy use on each problem was a better predictor of percentage of errors on that problem than were any of the other variables that we included in the regression analysis: the sum, the larger number, the smaller number, the first number, the second number, the square of the sum, or the min model. (For details of this and subsequent regression analyses, see Appendix A.)

The relation between overt strategy use and errors was not a simple causal one in which use of overt strategies caused children to err. Viewing each problem individually, on 24 of the 25 problems children erred on a lower percentage of trials on which they used overt strategies than on trials on which they did not, t(24) = 6.87, p < .01. For example, on the problem 3 + 4, children erred on 31% of trials on which they used overt strategies versus 80% of trials on which they did not.

Children’s use of overt strategies was also closely related to a second measure of problem difficulty, mean solution times. The longer the mean solution time on a problem, the higher the percentage of overt strategy use on that problem (r = .90). The relation could not be explained solely as the overt strategies’ taking longer to execute (although they did). Even when we excluded from our calculation of mean solution times those trials on which children used overt strategies, the relation remained substantial. Specifically, the correlation between mean

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Table 9.1

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Trials on Which Strategy Used (%)</th>
<th>Mean Solution Time (Sec)</th>
<th>Correct Answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting fingers</td>
<td>15</td>
<td>4.0</td>
<td>87</td>
</tr>
<tr>
<td>Fingers</td>
<td>13</td>
<td>6.6</td>
<td>89</td>
</tr>
<tr>
<td>Counting</td>
<td>8</td>
<td>9.0</td>
<td>54</td>
</tr>
<tr>
<td>Retrieval</td>
<td>64</td>
<td>4.0</td>
<td>66</td>
</tr>
</tbody>
</table>

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1No children used the counting or the counting-fingers strategy on the problem 1 + 1, thus leading to 23 rather than 24 degrees of freedom on comparisons involving those strategies.
solution times on retrieval trials on each problem and percentage of overt strategy use on that problem was $r = .76$. 2

To summarize, children used four strategies: fingers, counting fingers, counting, and retrieval. Retrieval was the fastest strategy, fingers the next fastest, and counting and counting fingers the slowest. Use of the overt strategies helped children solve problems. On 24 of the 25 problems, children added more accurately on trials on which they used overt strategies than on trials on which they did not. Finally, strong relations among the percentage of errors, the mean solution time, and the percentage of overt strategy use on each problem indicated that children had some systematic way of choosing when to use overt strategies.

We next consider how children might have arrived at their strategies.

### A DISTRIBUTION OF ASSOCIATIONS MODEL OF STRATEGY CHOICE

Fig. 9.2 outlines a model of how children generated their addition performance. We have labeled it the distribution of associations model, because within it errors, solution times, and overt strategy use are all functions of a single variable: the distribution of associations between problems and potential answers.

The model includes a representation and a process. The representation consists of associations of varying strengths between each problem and possible answers to the problem. The numerical values in the Fig. 9.2A matrix are the estimated strengths of these associations. For example, an associative strength of .05 links the problem $1 + 1$ and the answer "1," and an associative strength of .86 links $1 + 1$ and "2."

The process that operates on this representation can be divided into three phases: retrieval, elaboration of the representation, and counting. As shown in Fig. 9.2B, the child (who we here imagine as a boy) first retrieves an answer. If he is sufficiently confident of it, he states it. Otherwise, he next generates a more elaborate representation of the problem, perhaps by putting up fingers, and tries again to retrieve an answer. As before, if he is sufficiently confident of the answer he states it. Otherwise, he counts the objects in the representation and states the last number as the answer.

Now we can examine the process in greater detail. The first phase (Steps 1 to 8) involves an effort at retrieval. The child sets two parameters: a confidence criterion and a search length. The confidence criterion defines a value that must be exceeded by the associative strength of a retrieved answer for the child to state that answer. The search length indicates the maximum number of retrieval efforts the child will make before moving on to the second phase of the process. Once

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2 These estimated strengths were derived from performance in a separate "overt-strategies-prohibited" experiment. In this experiment, 4-year-olds were presented the Siegler and Robinson procedure except that they were explicitly asked to "just say what you think the right answer is without putting up your fingers or counting." The purpose of these instructions was to obtain the purest possible estimate of the strengths of associations between problems and answers. Each associative strength in the Fig. 9.2A matrix corresponds to the proportion of trials on which children advanced the particular answer to the particular problem in the overt-strategies-prohibited experiment.
these parameters are set, the child retrieves an answer. The probability of any given answer's being retrieved on a particular retrieval effort is proportional to the associative strength of that answer for that problem. Thus, the probability of retrieving "2" as the answer to "1 + 1" would be .86 (Fig. 9.2A). If the associative strength of the retrieved answer exceeds the confidence criterion, the child states that answer. Otherwise, the child examines whether the number-of

A. Representation (Associative Strengths)

<table>
<thead>
<tr>
<th>PROBLEM</th>
<th>ANSWER</th>
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<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1 + 1</td>
<td>.05</td>
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<tr>
<td>1 + 2</td>
<td>.09</td>
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<tr>
<td>1 + 3</td>
<td>.02</td>
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<tr>
<td>1 + 4</td>
<td>.11</td>
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<td>1 + 5</td>
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<td>2 + 1</td>
<td>.07</td>
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<td>2 + 2</td>
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<td>2 + 3</td>
<td>.04</td>
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<td>2 + 4</td>
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<td>5 + 4</td>
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<td>5 + 5</td>
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</table>

FIG. 9.2. The strategy choice model. In Fig. 9.2B, "answer," refers to whichever answer is retrieved on the particular retrieval effort. Also in Fig. 9.2B "problem-answer, associative strength" refers to the association between the elaborated representation and the retrieved answer.

searches that have been conducted is within the permissible search length. If so, the child again retrieves an answer, compares it to the confidence criterion, and advances it as the solution if its associative strength exceeds the criterion. Retrieval efforts continue as long as the associative strength of each retrieved answer is below the confidence criterion and the number of searches does not exceed the search length. If the point is reached at which the number of searches does exceed the search length, the child proceeds to the second phase.
In the second phase, the child creates an elaborated representation of the problem. This can be either an elaborated external representation, for example one in which the child puts up his fingers, or an elaborated internal representation, for example one in which the child forms a mental image of objects corresponding to augend and addend. Putting up fingers or forming an image adds visual associations between the elaborated representation and various answers to the already-existing association between the problem and various answers. If the elaborated representation involves the child's fingers, it adds kinesthetic associations as well. We refer to these visual and kinesthetic associations as elaborated representation-answer associations as opposed to the problem-answer associations discussed previously. Having formed the elaborated representation, the child again retrieves an answer.\(^4\) If that answer's associative strength exceeds the confidence criterion, the child responds. If it does not, the child proceeds to the third phase, an algorithmic process in which he or she counts the objects in the elaborated representation and advances the number assigned to the last object as the sum.

It may be useful to examine how a child using the model would solve a particular problem. Suppose a girl was presented the problem "3 + 4." Initially, she chooses a confidence criterion and a search length. For purposes of illustration, we assume that she selects the confidence criterion .50 and the search length two. Next, she retrieves an answer. As shown in Fig. 9.2A, the probability of retrieving 3 is .05, the probability of retrieving 4 is .11, the probability of retrieving 5 is .21, and so on. Suppose that the child retrieves 5. This answer's associative strength, .21, does not exceed the current confidence criterion, .50. Therefore, the girl does not state it as the answer. She next checks whether the number of searches has reached the search length. Because it has not, she again retrieves an answer. This time she might retrieve 7. The associative strength of 7, .29, does not exceed the confidence criterion, .50. Because the number of searches, two, has reached the allowed search length, the child proceeds to the second phase of the process.

In this second phase, the girl initially represents the problem either by forming a mental image or by putting up fingers. We assume that she puts up three fingers on one hand and four on the other. Next, she again retrieves an answer. As indicated in Footnote 4, combining the problem-answer and the representation-answer associative strengths increases the child's probability of retrieving 7 from .29 to .32. Suppose that she retrieves 7. Its associative strength still does not exceed the .50 confidence criterion. Therefore, the child does not state it. She instead proceeds to the third phase of the process. Here, she counts her fingers and states the last number as the answer to the problem. If she counts correctly, she will say "7."

This model accounts for the strategies that children use, for the temporal characteristics of the strategies, and for the close relations among the percentage of overt strategy use, the percentage of errors, and the mean solution times on each problem. First consider how it accounts for the existence of the four strategies. The retrieval strategy appears if children retrieve an answer whose problem-answer associative strength exceeds their confidence criterion (Steps 1 to 5, sometimes Steps 6 and 7, Step 8). The fingers strategy emerges when children fail to retrieve an answer whose problem-answer associative strength exceeds their confidence criterion, put up their fingers, and then retrieve an answer in which the sum of the problem-answer and the elaborated representation-answer associative strengths exceeds their confidence criterion (Steps 1 to 7, 9 to 10, 12 to 14). The counting-fingers strategy appears if children fail to retrieve an answer whose problem-answer associative strength exceeds their confidence criterion, put up their fingers, fail to retrieve an answer in which the sum of the elaborated representation-answer and problem-answer associative strengths exceeds the confidence criterion, and finally count their fingers (Steps 1 to 7, 9 to 10, 12 to 13, 15 to 17). The counting strategy is observed if children fail to retrieve an answer whose problem-answer associative strength exceeds their confidence criterion, form an elaborated internal representation, fail to retrieve an answer in which the sum of the elaborated representation-answer and problem-answer associative strengths exceeds the confidence criterion, and finally count the objects in the internal representation (Steps 1 to 7, 9, 11 to 13, 15 to 17).\(^5\)

The relative solution times of the strategies arise because the faster strategies are component parts of the slower ones. To use the fingers strategy, children must execute all of the steps in the retrieval strategy and four additional ones. To execute the counting-fingers strategy, children must proceed through all of the steps in the fingers strategy and two additional ones. To execute the counting strategy, children must execute all of the steps in the retrieval strategy plus six others. If we can equate the time needed to form elaborated internal and elaborated external representations, children using the counting strategy must execute all of the steps in the fingers strategy plus two others. Thus, the retrieval strategy should be faster than any of the other strategies, the fingers strategy should be

\(^4\) The probability of a given answer's being retrieved at this point is determined by adding the problem-answer and the elaborated representation-answer associative strengths and dividing by one plus the elaborated representation-answer associative strength. In our computer simulation (described later), we arbitrarily decided that each external representation added a constant .05 to the answer corresponding to the number of objects in the representation. Thus, if the problem 1 + 4 was represented with five fingers, and the initial associative strength of 1 + 4 = 5 was .61, the new associative strength would be .66/1.05 = .63.

\(^5\) Any trials on which children generated an internal representation and then stated the answer that they retrieved (Steps 1 to 7 and 10 to 13) would also be classified as retrieval trials. This path was expected to be rare, however. Kinesthetic cues would not be available to mediate the elaborated representation-answer association, and visual cues would be weaker than if the objects in the elaborated representation were visible.
faster than the counting-fingers strategy, and, if the time needed to form an external representation does not exceed the time needed to form an internal one, the fingers strategy also should be faster than the counting strategy.²

Perhaps the most important feature of the model is that it generates close associations among percentage of errors, mean solution time, and percentage of overt strategy use on each problem. The associations arise because all three dependent variables are functions of the same independent variable: the distribution of associations linking problems and answers. The way in which this dependency operates becomes apparent when we compare the outcomes of a peaked distribution of associations, such as that for 2 + 1 in Fig. 9.3, with those of a flat distribution, such as that for 3 + 4. A low percentage of use of overt strategies, a low percentage of errors, and a short mean solution time all accompany the peaked distribution. Relative to the flat distribution, the peaked distribution results in: (1) less frequent use of overt strategies (because the answer that is retrieved is more likely to have high associative strength, which allows it to exceed more confidence criteria, thus leading to use of retrieval rather than overt strategies); (2) fewer errors (because of the higher probability of retrieving and stating the answer that forms the peak of the distribution, which is generally the correct answer); and (3) shorter solution times (because the probability of retrieving on an early search an answer whose associative strength exceeds any given confidence criterion is greater the more peaked the distribution of associations).

At least two nonintuitive predictions follow from this model. One is that the correlation between percentage of errors on each problem and percentage of overt strategy use on that problem is primarily a correlation between the percentage of errors on retrieval trials on each problem and the percentage of overt strategy use on the problem. That is, the correlation does not depend on the percentage of errors on counting, counting-fingers, and fingers strategy trials. The reasoning underlying this prediction is that percentage of overt strategy use on a problem and percentage of errors on retrieval trials on that problem both depend entirely on the distribution of associations, but percentage of errors on nonretrieval trials on the problem depends on other factors.

This logic may become clearer when we examine the model’s account of the way in which errors are produced by each of the four strategies. On retrieval trials, the percentage of errors on each problem depends only on the distribution of associations. Errors are made when an incorrect answer retrieved from the distribution exceeds the confidence criterion. The flatter the distribution, the greater the proportion of retrieval trials on which this will happen. The distribution’s flatness increases both the likelihood of an incorrect answer’s being retrieved and the likelihood of its having sufficient associative strength to be stated. Thus percentage of errors on retrieval trials, like percentage of overt strategy use, depends entirely on the distribution of associations.

In contrast, the percentage of errors on counting and counting-fingers trials is unaffected by the distribution of associations. The counting and counting-fingers strategies arise when children fail to retrieve a storable answer from the distribution of associations. Instead, they base answers on their counts of the objects in their elaborated representations. Errors are made when they misrepresent the number of objects in the problem or when they miscount them. The greater the sum, the more objects that can be misrepresented or miscounted, and therefore the greater the likelihood of errors. Thus, colinearity with the sum should account for whatever correlation emerges between the percentage of errors on counting and counting-fingers trials on each problem and the percentage of overt strategy use on the problem.

Correlations involving the percentage of errors on fingers-strategy trials should occupy a middle ground. Recall that the fingers strategy is produced when children first fail to retrieve an answer whose associative strength exceeds their confidence criterion, then elaborate the representation by putting up their fingers, and then retrieve an answer in which the sum of the problem-answer and the elaborated representation-answer associative strengths exceeds the confidence criterion. Under such circumstances, the percentage of errors is a function of both the distribution of associations between problems and answers and the

²The relative accuracy of the four strategies can also be explained within the framework of the model, though the explanation involves several considerations external to the model. See Siegler and Robinson (1982, pp. 298–299) for this explanation.
distribution of associations between elaborated representations and answers. A relatively peaked distribution of associations increases the probability that the problem-answer and elaborated representation-answer associations together will lead to retrieval of an answer whose associative strength exceeds the confidence criterion. However, the sum also may influence this likelihood. Presumably, children more often correctly represent the addends on problems with small sums, which leads to the elaborated representation-answer association’s more often being added to the correct rather than to an incorrect answer on these problems. Thus, the percentage of errors on fingers-strategy trials on each problem should correlate somewhat with percentage of overt strategy use on that problem, but not as highly as percentage of errors on retrieval trials on the problem.

The logic of these predictions can be summarized as follows:

A. If percentage of overt strategy use on each problem is a function of only the distribution of associations; and
B. If percentage of errors on retrieval trials on each problem is also a function of only the distribution of associations; and
C. If percentage of errors on fingers trials is in part a function of the distribution of associations; and
D. If percentage of errors on counting and counting-fingers trials is not at all a function of the distribution of associations, instead being a function of the sum;

Then

1. Percentage of errors on retrieval trials should correlate highly with percentage of overt strategy use.
2. Percentage of errors on counting and counting-fingers trials should correlate less highly with percentage of overt strategy use, especially when the contribution of the sum is partialed out.
3. Percentage of errors on fingers trials should show an intermediate degree of correlation with percentage of overt strategy use.

The data were entirely consistent with these predictions. As shown in Fig. 9.4, percentage of errors on retrieval trials on each problem correlated \( r = .92 \) with percentage of overt strategy use on the problem. This correlation was actually slightly higher than the correlation reported earlier between the percentage of errors on all trials and the percentage of overt strategy use. The correlation between percentage of errors on counting and counting-fingers trials on each problem and the percentage of overt strategy use on that problem \( (r = .38) \) was significantly lower, \( t(22) = 6.47, p < .01 \). Yet more striking was the difference between the partial correlations. When the contribution of the sum was partialed out, percentage of overt strategy use correlated \( r = .87 \) with percentage of errors on retrieval trials. The corresponding partial correlation between percentage of
overt strategy use and percentage of errors on counting and counting-fingers trials was \( r = -0.23 \). The difference between the two partial correlations was highly significant \( t(22) = 7.45, p < .01 \).

Also as predicted, the correlation between percentage of errors on fingers trials and percentage of overt strategy use on each problem was in between \( (r = .68) \). It did not differ significantly from either of the other correlations. When the contribution of the sum was partialed out, the correlation became \( r = .66 \). This correlation was significantly lower than the one involving errors on retrieval trials \( t(22) = 2.10, p < .05 \) and significantly higher than the one involving errors on counting and counting-fingers trials \( t(22) = 5.08 \). The findings supported the view that the original correlation between errors and overt strategy use was due primarily to overt strategy use and errors on retrieval trials being functions of the same variable, the distribution of associations, and of errors on overt strategy trials depending on other variables.

Similar logic can be applied to analyzing the correlation between solution times and overt strategy use. Solution times on retrieval trials should derive exclusively from the distribution of associations. The more peaked this distribution, the more quickly children should retrieve an answer whose associative strength exceeds their confidence criterion. On counting and counting-fingers trials, the amount of time needed to generate an elaborated representation and to count the objects in it would influence the times. These would depend on the number of objects that need to be represented and counted—in short, on the sum. The model made no direct prediction concerning solution times on fingers trials on each problem. Thus, the prediction of the model was that the correlation between percentage of overt strategy use and mean solution times on retrieval trials on each problem would be higher than the correlation between percentage of overt strategy use and mean solution times on counting and counting-fingers trials. The pattern would be most evident with the contribution of the sum partialed out from both correlations.

A multiple regression analysis indicated that, as predicted by the model, percentage of overt strategy use was the most powerful predictor of mean solution times on retrieval trials on the 25 problems \( (r = .76) \) (Appendix A). Contrary to expectation, however, the best predictor of solution times on counting and counting-fingers trials on each problem was also percentage of overt strategy use \( (r = .83) \). The partial correlations showed the same pattern. With the contribution of the sum partialed out, the correlation between percentage of overt strategy use on each problem and mean solution times on retrieval trials on the

---

7This finding argued against the possibility that the difference in the magnitudes of the correlations involving errors on retrieval trials and those involving errors on counting and counting-fingers trials was due to the latter’s being based on fewer trials per problem. If trials per problem were the key variable, we would not expect percentage of overt strategy use to correlate more highly with the percentage of errors on fingers trials than with the percentage of errors on counting and counting-fingers trials. There were fewer fingers trials than counting and counting-fingers trials.

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8As becomes evident later, patterns of solution times in all subsequent experiments conformed to the predictions of the model. Therefore, no effort was made to explain why the solution times in this experiment did not.

9. STRATEGY CHOICES IN ADDITION AND SUBTRACTION

The problem was \( r = .68 \). The corresponding partial correlation between overt strategy use and solution times on counting and counting-fingers trials was \( r = .71 \).

In summary, all but one of the model’s predictions were consistent with the data. The model accounted for the four strategies that children used, the relative solution times of the strategies, the correlations among percentage of errors, mean solution times, and percentage of overt strategy use on each problem, and the source of at least the correlation between errors and strategy use’s being percentage of errors on retrieval trials. The model also possessed several other properties that seemed desirable. It allowed children to strike a balance between speed and accuracy demands. When possible, they would use the relatively rapid retrieval approach. When retrieval yielded no answer that was sufficiently strongly associated with the problem, they would fall back on successively more time-consuming overt approaches. The model also had the advantage of treating all problems in the same way. It did not assume that ties have a special status or that the mental distance between sums increases exponentially with their sizes. Finally, as is discussed in more detail later in the chapter, the model suggests how development might occur. As children’s distributions of associations become increasingly peaked, they rely increasingly on retrieval, advance the correct answer more often, and answer more quickly. In short, their performance becomes increasingly adultlike.

A MATHEMATICAL EXPRESSION OF THE MODEL

We wanted to provide a rigorous test of the sufficiency of the model to produce strong relations among overt strategy use, frequency of errors on retrieval trials, and length of mean solution times on each problem. Therefore, we translated the model’s predictions into algebraic equations, inserted a large range of parameter values into the model, and compared the model’s behavior to that that children had displayed.

The following equations were used to describe, for each problem, the probability of retrieving an answer that exceeded the confidence criterion, the probability of overt strategy use, the probability of an error on a retrieval trial, and the expected solution time on a retrieval trial.

Probability of Retrieving Answer on a Problem that Exceeds Confidence Criterion:

\[
R = \frac{\sum_{a=1}^{A} (AS_a)(p(AS_a > CC))}{\sum_{a=1}^{A} AS_a}
\]
Probability of Overt Strategy Use on a Problem = \((1 - R)^N\)

Probability of Error on Retrieval Trials on Each Problem =

\[ 1 - ((p(AS_{cs} > CC))(AS_{cs})/\sum_{a=1}^{A} (AS_a(p(AS_a > CC)))) \]

Expected Solution Time on Retrieval Trials on Each Problem =

\[ \sum_{n=1}^{N} nR((1 - R)^{n-1}) + N(1 - \sum_{n=1}^{N} R((1 - R)^{n-1})) \]

where \( R \) refers to the probability that the answer retrieved on any given search will exceed the confidence criterion, \( AS_{cs} \) refers to the associative strength of answer \( a \), \( CC \) refers to the confidence criterion, \( N \) refers to the search length, and \( AS_{ca} \) refers to the associative strength of the correct answer.

The correspondence between each equation and the process it models is quite straightforward. The probability on each problem of retrieving an answer that exceeds the confidence criterion is the sum of the associative strengths of answers to that problem that exceed the criterion divided by the sum of the associative strengths of all answers to the problem. The probability of overt strategy use on a problem is the probability that on none of the searches will an answer be retrieved that exceeds the confidence criterion. The probability of an error on a retrieval trial is the probability of retrieving an incorrect answer whose associative strength exceeds the confidence criterion divided by the probability of retrieving a correct or incorrect answer whose associative strength exceeds the confidence criterion. The expected solution time on retrieval trials is proportional to the expected value of the number of searches on each problem before an answer is stated.

We examined the operation of this mathematical model under the 72 possible combinations of confidence criteria (.05, .10, .15, .20, .30, .40, .50, .60, .70, .80, .90, 1.00) and search lengths (one to six). For each confidence-criterion–search-length pair, we applied the four equations to each of the 25 problems that children had been presented. Then we combined the results to obtain expected percentages of errors on retrieval trials, mean solution times on retrieval trials, and percentages of overt strategy use.

The model was tested in two ways, corresponding to measures of internal and external validity. First we wanted to establish the sufficiency of the equations to generate the high correlations among the three variables that we had observed. To do this, we entered into the equations associative strengths (operationally defined here as the relative frequencies of answers given on retrieval trials in Siegler and Robinson) and used the output of the equations to estimate percentages of errors, mean solution times, and percentages of overt strategy use on each problem. If the equations operated as anticipated, this procedure would produce high intercorrelations among the expected values for the three variables. The equations passed this test. The intercorrelations among the output of the equations for errors, solution times, and overt strategy use ranged from \( r = .92 \) to \( r = .99 \).

As another measure of internal validity, we tested whether the model’s predictions for each measure would correlate highly with the children’s mean solution times, percentage of overt strategy use, and percentage of errors in the Siegler and Robinson experiment. The correlation between modeled and observed percentage of errors on retrieval trials was \( r = .94 \), between modeled and observed frequency of overt strategy use \( r = .89 \), and between modeled and observed mean solution times on retrieval trials \( r = .92 \).

We next tested the mathematical model’s ability to predict across data sets. We used the associative strengths displayed in Fig. 9.2A to predict performance in the Siegler and Robinson experiment. As indicated in Footnote 2, these associative strengths were estimated from the performance of different children under somewhat different experimental conditions (no overt strategies allowed) than the data being predicted. The correlation between the predicted and observed percentage of overt strategy use on each problem was \( r = .87 \). The correlation between the predicted and observed percentages of errors on each problem was \( r = .77 \). The correlation between the predicted and observed mean solution times on each problem was \( r = .83 \). These results demonstrated that estimates of associative strengths obtained in one experiment could be used to predict experimental data in another.

**A REPLICATION AND EXTENSION EXPERIMENT**

Many results from the initial experiment were unanticipated. Also, it seemed possible that the generality of the findings would be limited to problems in which the fingers and counting-fingers strategies were easy to use—that is, problems with addends no greater than 5 and/or sums no greater than 10. We therefore performed a second experiment replicating the initial condition and adding problem sets on which overt strategies would be more difficult to execute.

A further purpose of the experiment was to test an alternative to the Fig. 9.2 model of strategy choice. In this alternative, the close connection between errors, solution times, and overt strategy use arises because children explicitly judge each problem’s difficulty and use an overt strategy when they judge the difficulty to be high. Thus:

- problem difficulty → judgments of problem difficulty → overt strategy use

This depiction suggests that judgments of problem difficulty should correlate highly with both actual problem difficulty (as measured by percentage of errors
on each problem) and overt strategy use. Otherwise the observed high correlation between percentage of errors and overt strategies would be difficult to explain.

In a preliminary test of this alternative, Siegler and Robinson (1982) asked a group of 5 year olds, students at a nursery school similar to the one at which the original experiment had been run, to label each of the 25 problems as hard, easy, or in between. “Hard” ratings were quantified as 2, “easy” ratings as 0, and “in-between” ratings as 1. It was found that the mean difficulty ratings on each of the 25 problems correlated $r = .47$ with the percentage of errors on the problem and $r = .51$ with the percentage of overt strategy use on the problems. These correlations were substantially lower than the previously noted $r = .91$ correlation between percentage of overt strategy use on each problem and percentage of errors on the problem. Because the judgment data were collected from different children than the addition performance data, however, the experiment provided only a preliminary index. The replication and extension experiment, in which both types of data were obtained from the same children, would provide a more definitive test.

**Method**

The 42 children who participated, 23 boys and 19 girls, attended either a university preschool or a nursery school in a middle-class area of Pittsburgh. In each of the three conditions, there were eight 4 year olds and six 5-year-olds.

Children in Group 1 of this experiment (the replication condition) were presented the same 25 problems as the children studied by Siegler and Robinson. These were all of the problems on which both addends were less than or equal to 5 and on which the sum was less than or equal to 10. Children in Group 2 were presented 25 problems on which the sum again was less than or equal to 10 but on which either addend could be as great as 9. Children in Group 3 were presented a third set of 25 problems; in this set, the sum could be as high as 12 and the addends as large as 11. To maintain the interest of children who were not especially skillful at adding, Groups 2 and 3 included two subsets of problems. In each group, 13 of the problems were selected from the relatively easy items presented in Group 1 in which addends never exceeded 5 and sums never exceeded 10. The remaining 12 problems in Group 2 all had one addend between 6 and 9 and a sum that did not exceed 10. The remaining 12 problems in Group 3 all had one addend between 6 and 11 and sums of 11 or 12.

The instructions began in the same way as those used by Siegler and Robinson. However, to ensure that all children knew that they could use overt strategies, we added the following instructions at the end: “You can do anything you want to help you get the right answer. If you want to use your fingers or count aloud, that’s fine.” It seemed possible that these instructions would increase overall use of the overt strategies, but unlikely that they would influence the correlation between use of overt strategies and problem difficulty.

Within a week of their last counting session, children in all three groups were presented the metacognitive judgment task that peers in the Siegler and Robinson study had performed. They were told, “Remember the problems I asked you about? Well, some of those problems were easy, some were hard, and some were in between easy and hard. I’m going to ask you about each problem and you tell me whether you think that it is easy, hard, or in between.”

**Results**

**Existence of Strategies.** The model suggested that children would use four strategies: counting fingers, fingers, counting, and retrieval. As shown in Table 9.2, children in all groups used each of these approaches. We had anticipated that the new instructions’ explicit statement that children could use overt strategies might result in more frequent use of such strategies. The data showed little tendency in this direction, however. In the Siegler and Robinson study, children used overt strategies on 36% of trials; in the replication and extension experiment, they used them on 43%. These percentages did not differ significantly, $t < 1$.

Children in Group 3 encountered items on which the sum exceeded their number of fingers. This situation appeared to lead them to adopt new procedures, because on 8% of trials they used strategies that did not fall neatly into the four categories. The two most common variants, each accounting for 3% of total trials, were counting/counting fingers on and counting fingers/fail. In the first of these, children would count up to one addend without putting up their fingers, then put up their fingers to represent the other addend, and then count their fingers starting with the number one greater than the result of their initial counting procedure. As long as the addend that children represented on their fingers was no greater than 10, this procedure circumvented the difficulty of the sum’s exceeding their number of fingers. The second relatively common procedure involved representing as many numbers as the child had fingers, counting all of them, and then arbitrarily naming a larger number if both addends had not been totally represented. Several other procedures were used very occasionally, none of them exceeding 1% of total trials. Like the procedures just described, these

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*A variant of the counting strategy that was not observed by Siegler and Robinson appeared in this experiment. On some trials, children put up fingers synchronously with counting out the sum. That is, they put up a first finger while saying "1," a second finger while saying "2," and so on. These trials could not be classified as counting-fingers trials, because there was no evidence that the fingers were being used to represent the addends. Unlike the trials that were labeled counting-fingers, children did not represent the addends separately from counting out the sum, did not pause between addends, and did not represent addends on separate hands. Accuracy rates on these trials were also more similar to those observed on counting than on counting-fingers trials. Therefore, these approaches were viewed as a form of counting in which fingers were incidental accompaniments, much as tapping one’s feet might be.*
involved combining the usual strategies in innovative ways, and appeared aimed at overcoming the difficulty associated with sums greater than 10.

Relative Accuracy of Strategies. The relative accuracy of the four strategies was identical to that found in the earlier study. In all three groups, the fingers strategy was the most accurate, the counting-fingers strategy the next most accurate, and the retrieval and counting strategies the least accurate. Also as previously, children performed more accurately on those trials on each problem on which they used overt strategies than on those trials on which they did not (for Group 1, \( t(24) = 5.14 \); for Group 2, \( t(24) = 2.89 \); for Group 3, \( t(24) = 3.61 \); all \( p's < .01 \)). Thus, using the overt strategies seemed to help children solve the problems.

Relative Solution Times of Strategies. The model predicted that retrieval would be the fastest strategy, that fingers would be the next fastest, and that counting and counting fingers would be the slowest. The data were in accord with these predictions. As shown in Table 9.2, the retrieval and fingers strategies were substantially faster than the counting and counting-fingers strategies in each of the three groups. Including those children from all three groups who used both of the pairs of strategies at least twice, retrieval was significantly faster than the fingers strategy, \( t(18) = 2.34, p < .05 \), which in turn was significantly faster than the counting strategy, \( t(14) = 2.42, p < .05 \), or the counting-fingers strategy, \( t(13) = 3.53, p < .01 \).10

Relations among Errors, Solution Times, and Overt Strategy Use. The model predicted that the percentage of errors, mean solution times, and percentage of overt strategy use on each problem would vary together. The expected pattern emerged in all three groups. In Group 1, percentage of overt strategy use on each problem correlated \( r = .79 \) with percentage of errors on that problem. In Group 2, the two variables correlated \( r = .79 \). In Group 3, they correlated \( r = .81 \). The relation is illustrated in Fig. 9.5 and in Appendix A.

A similar relation was present between overt strategy use and solution times. In Group 1, mean solution times correlated \( r = .91 \) with percentage of overt strategy use; in Group 2, the two variables correlated \( r = .81 \); and in Group 3, the two variables correlated \( r = .92 \). As in Siegler and Robinson, the relations were present even when only solution times on retrieval trials were considered: \( r = .80 \), \( r = .75 \), and \( r = .90 \) for Groups 1, 2, and 3, respectively (Appendix A).

The Source of the Relations among Strategy Use, Errors, and Solution Times. The model predicted that the correlations among these three variables derived primarily from percentage of errors and mean solution times on retrieval trials. This prediction again proved accurate. First consider analyses involving errors on retrieval trials. In each of the three groups, the best predictor of the percentage of errors on retrieval trials on each problem was the percentage of overt strategy use on that problem (Appendix A). The correlations ranged from \( r = .80 \) to \( r = .88 \) (Table 9.3). In all three cases, as in the Siegler and Robinson data, these correlations were greater than the correlations involving the overall percentage of errors. When the contribution of the sum was partialled out, the correlation between errors on retrieval trials and overt strategy use remained quite high, ranging from \( r = .56 \) to \( r = .70 \). The correlations involving percentage of errors on counting and counting-fingers trials showed a different pattern. In none of the three groups was overt strategy use the best predictor of percentage of errors on these trials. The raw correlations ranged from \( r = .37 \) to \( r = .51 \). With the contribution of the sum partialled out, the correlations ranged from \( r = .23 \) to \( r = .42 \).

For each of the three groups, the raw correlation between percentage of errors on retrieval trials and percentage of overt strategy use was significantly greater than the corresponding correlation between percentage of errors on counting and counting-fingers trials and percentage of overt strategy use. The difference be-

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10The reason for comparing the solution times of children in all three groups in a single analysis, rather than comparing the times in each group separately, was to obtain enough children who used each pair of strategies to make a statistical comparison reasonably powerful. In many cases, only five or six children in each group used a particular pair of strategies.
TABLE 9.3
Source of Correlations of Overt Strategy Use with Errors and Solution Times: Replication and Extension Experiment

<table>
<thead>
<tr>
<th>A. Raw Correlations</th>
<th>Solution Times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Retrieval Trials</td>
</tr>
<tr>
<td></td>
<td>Fingers Trials*</td>
</tr>
<tr>
<td>Group 1</td>
<td>.83</td>
</tr>
<tr>
<td>Group 2</td>
<td>.80</td>
</tr>
<tr>
<td>Group 3</td>
<td>.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Partial Correlations (Correlations with the Sum Partialed Out)</th>
<th>Solution Times</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Retrieval Trials</td>
</tr>
<tr>
<td></td>
<td>Fingers Trials*</td>
</tr>
<tr>
<td>Group 1</td>
<td>.67</td>
</tr>
<tr>
<td>Group 2</td>
<td>.56</td>
</tr>
<tr>
<td>Group 3</td>
<td>.70</td>
</tr>
</tbody>
</table>

*Numbers in the Table indicate correlations of percentage of overt strategy use on each problem with the variable specified. For example, the top left value of .83 indicates a raw correlation of $r = .83$ between percentage of errors on retrieval trials on each problem and percentage of overt strategy use on that problem for Group 1.
*Summing across the performance of children in all three groups, the correlation was $r = .68$.
<Summing across the performance of children in all three groups, the correlation was $r = .66$.}

The source of the correlation between solution times and overt strategy use was also in accord with the prediction of the model. In all three groups, the best predictor of mean solution times on retrieval trials on each problem was percentage of overt strategy use on that problem (Appendix A). In contrast, in all three groups the sum was a better predictor of mean solution times on counting and counting-fingers trials on each problem than was the percentage of overt strategy use on that problem. The differences in the magnitudes of the raw correlations was significant for Group 1, $t(22) = 2.60, p < .05$, though not for the other two groups, $t(22) = 1.45$ and $.84$. The comparisons of the partial correlations yielded more striking results. As shown in Table 9.3, large differences separated the magnitudes of the partial correlations involving solution times on retrieval and on counting and counting-fingers trials. The differences were significant for all three groups ($t(22) = 3.38, 2.24$, and $5.07$ for Groups 1, 2, and 3, respectively, all $p$'s $< .05$).

**Explicit Judgment Data.** Explicit judgments of problem difficulty collected from the same subjects who provided the addition performance data showed a similar pattern to that previously reported with between-subjects data. The correlations between mean rating of problem difficulty for each problem and percentages of errors on that problem were $r = .64$, $r = .34$, and $r = .70$ for Groups 1, 2, and 3, respectively. The correlations between mean rating of difficulty for each problem and percentage of overt strategy use on that problem were $r = .73$, $r = .50$, and $r = .53$, respectively. All six of these correlations were lower than any of the three correlations between percentages of overt strategy use and errors. Four of the six were also lower than the corresponding correlations between the metacognitive judgments and the size of the larger addend, suggesting that it was not unreliability of measurement of the metacognitive judgments that led to the lower correlations. The slippage that would be entailed in going from problem difficulty to judgments of problem difficulty and then from judgments of problem difficulty to use of overt strategies added to the unlikelihood that the accuracy of the metacognitive judgments of difficulty could account for the correlations between overt strategy use and errors.

**Summary of Replication and Extension Experiment Findings.** The results of the replication and extension experiment indicated that the earlier results were not due to any peculiarity of the problem set. The data from each of the three groups closely paralleled the Siegler and Robinson findings concerning the existence of the four strategies, their relative accuracy, their relative solution times, the correlations among percentage of overt strategy use, mean solution time, and percentage of errors on each problem, and the source of these correlations being
errors and solution times on retrieval trials. On the new problems, as on the original ones, young children demonstrated the ability to make adaptive strategy choices even without the ability to make highly accurate judgments about problem difficulty.

AN EXPERIMENT ON SUBTRACTION

Although the model was developed to account for strategy choices on addition problems, it seemed to provide a plausible model of strategy choices on simple subtraction problems as well. To use the counting-fingers strategy, children could raise fingers to represent the larger number, lower fingers corresponding to the smaller number, and count the remaining fingers. To use the fingers strategy, they could put up fingers representing the larger number, put down fingers representing the smaller number, and answer without counting the remainder. To use the counting strategy, they could count backward from the larger number, or count up to it from one and then count backward, without putting up fingers. To use the retrieval strategy, they could retrieve an answer and state it without any intervening visible or audible behavior. The expectations for the relative solution times of the strategies, the relations among percentage of overt strategy use, mean solution times, and percentage of errors on each problem, and the source of these relations being errors and solution times on retrieval trials would be the same as in addition. The subtraction experiment was performed to test whether subtraction did resemble addition in these ways.

Method

Participants were 34 children, half 5-year-olds and half 6-year-olds, half of each age group boys and half girls. The 5-year-olds attended a university preschool. The 6-year-olds attended the first grade of an upper-middle-class suburban school.

The problems were the inverses of the 25 addition problems presented by Siegler and Robinson. For every problem of the form \(a + b = c\) in the Siegler and Robinson study, here there was a problem of the form \(c - b = a\). The instructions were:

I want you to imagine that you have a pile of oranges. Then imagine that I take some oranges away from your pile. Tell me how many you have left. You can do anything you want to help you get the right answer. If you want to use your fingers or count aloud, that’s fine. Okay? Suppose you have \(m\) oranges, how many would you have if I took away \(n\) of them?

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Trials on Which Strategy Used (%)</th>
<th>Mean Solution Time (Sec)</th>
<th>Correct Answers (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting</td>
<td>21</td>
<td>9.8</td>
<td>83</td>
</tr>
<tr>
<td>Fingers</td>
<td>14</td>
<td>4.4</td>
<td>94</td>
</tr>
<tr>
<td>Counting</td>
<td>7</td>
<td>9.1</td>
<td>25</td>
</tr>
<tr>
<td>Retrieval</td>
<td>58</td>
<td>4.4</td>
<td>68</td>
</tr>
</tbody>
</table>

Results

Existence of Strategies. The videotapes revealed that children used the same four strategies as they had in the addition experiments. As indicated in Table 9.4, children used overt strategies on subtraction problems at least as often as peers previously had used them in adding. Also as previously, retrieval was the most frequently used strategy, being employed on 58% of trials, a figure comparable to the 64% and 57% in the previous studies. The relative accuracy of the four strategies was also identical to those obtained in the addition studies. Fingers was the most accurate strategy, counting fingers the next most accurate, and retrieval and counting the least accurate.

Relative Solution Times of Strategies. The relative solution times again followed the pattern predicted by the model. The retrieval strategy was significantly faster than the fingers strategy \(t(17) = 2.70\), which was significantly faster than either the counting strategy \(t(7) = 3.43\) or the counting-fingers strategy \(t(17) = 7.15\) (all \(p's < .05\)).

Relations among Errors, Solution Times, and Overt Strategy Use. As shown in Fig. 9.6, a strong relation again emerged between the percentage of errors on each problem and the percentage of overt strategy use on that problem. The correlation \((r = .83)\) was of the same magnitude that had appeared in the addition studies. As previously, percentage of overt strategy use on each problem correlated more highly with the error rate on the problem than did any of the other variables that were tested (remainder, remainder squared, larger number, smaller number, the Woods, Restick, and Groen (1975) subtraction model,\(^{11}\) or

\(^{11}\)In this subtraction model, children chose whichever of two strategies was easier to execute. They could either count down from the larger number the number of times indicated by the smaller, or they could count up from the smaller number the number of times needed to reach the larger, whichever took fewer counts. On 7 - 3, they would count down from 7 because it required three rather than four counts; on 7 - 4, they would count up from 4 for the same reason.
the sum) (Appendix A). The correlation between mean solution times on each problem and percentage of overt strategy use on that problem \(r = .88\) was also similar in magnitude to those obtained in the earlier experiments.

The Source of the Relations among Strategy Use, Errors, and Solution Times. The model predicted that the correlations among these three variables derived primarily from errors and solution times on retrieval trials. Errors and solution times on these trials, like overt strategy use, would derive from the distribution of associations. By contrast, errors and solution times on counting and counting-fingers trials would derive from the size of the numbers being counted. In the operation of addition the size of the numbers being counted corresponds to the sum. In subtraction, it corresponds most naturally to the size of the larger number. Illustratively, a child who is counting fingers first needs to represent the larger number, then to put down the number of fingers corresponding to the second number, and then to count the remainder. Adding the larger number, the smaller number, and the remainder yields a sum that is twice the larger number. Therefore, the percentage of overt strategy use seemed likely to be an accurate predictor of errors and solution times on retrieval trials in subtraction, but not of errors and solution times on counting and counting-fingers trials, especially when the contribution of the larger number was partialed out.

These predictions proved accurate. First consider the data on retrieval-trial errors. Percentage of errors on retrieval trials on each problem correlated \(r = .83\) with percentage of overt strategy use on the problem. When the contribution of the size of the larger number was partialed out, the correlation between percentage of errors on retrieval trials and percentage of overt strategy use remained high: \(r = .72\). In contrast, percentage of errors on counting and counting-fingers trials correlated only \(r = .25\) with percentage of overt strategy use (Fig. 9.6). With the contribution of the larger number partialed out, the correlation between errors on counting and counting-fingers trials and overt strategy use became \(r = .00\). On both the raw and the partial correlations, percentage of errors on retrieval trials was significantly more closely related to percentage of overt strategy use than was percentage of errors on counting and counting-fingers trials. For the raw correlations, the difference in the correlations was \(t(22) = 4.09\). For the partial correlations, the difference between the two correlations was \(t(22) = 3.51\) (both \(p < .01\)).

The model predicted that the correlation between percentage of errors on fingers-strategy trials and percentage of overt strategy use would be lower than the correlation of overt strategy use with errors on retrieval trials and higher than the correlation of overt strategy use with errors on counting and counting-fingers trials (for the same reason as on addition problems). The raw correlation between percentage of errors on fingers-strategy trials on each problem and percentage of overt strategy use on that problem was \(r = .39\), which was between the other two raw correlations. This correlation was significantly lower than the raw corre-
tion with errors on retrieval trials, \( t(22) = 3.51, p < .01 \), but not significantly different than the raw correlation with errors on counting and counting-fingers trials \( t < 1 \). With the contribution of the larger number partialled out, the correlation was \( r = .11 \). This partial correlation again was in between the other two. It was significantly lower than the partial correlation involving errors on retrieval trials, \( t(22) = 3.36, p < .01 \), and did not differ significantly from the partial correlation involving errors on counting and counting-fingers trials, \( t < 1 \).

A similar pattern emerged for solution times. Overt strategy use correlated highly with mean solution times on retrieval trials (\( r = .83 \)). The magnitude of the correlation was reduced somewhat but remained substantial when the magnitude of the larger number was partialled out (\( r = .62 \)). Solution times on counting and counting-fingers trials also correlated highly with overt strategy use (\( r = .73 \)). This correlation, however, was based largely on covariation with the larger number. When the contribution of the larger number’s size was partialled out, the correlation between percentage of overt strategy use and mean solution times on counting and counting-fingers trials fell to \( r = .25 \). The difference between the raw correlations did not reach significance, \( t(22) = 1.20, p > .10 \), but the difference between the partial correlations was significant, \( t(22) = 2.45, p < .05 \).

These results indicate that in subtraction, as in addition, the types of strategies that children use, the relative solution times of the strategies, the correlations among percentage of errors, mean solution times, and percentage of overt strategy use on each problem, and the source of the correlations’ residing in errors and solution times on retrieval trials all matched the pattern predicted by the distribution of associations model. Next we consider how young children might develop such an adaptive strategy-choice procedure.

THE DEVELOPMENT OF STRATEGY CHOICES

How might children acquire their distributions of associations, and how might they acquire a process for operating on them? First consider the distributions of associations themselves. A satisfactory acquisition theory would need to explain learning of both correct answers and errors, because children’s distributions of associations include incorrect as well as correct responses. Our basic assumption about how children acquire these distributions is that each time they answer a problem, the associative strength linking that answer to the problem increases. This idea has frequently been used to account for the learning of correct answers but has rarely been used to explain the learning of particular errors. A recent finding by Jacoby (personal communication), however, suggests that the mechanism may apply there as well. Jacoby found that showing college students words spelled incorrectly slowed their later recognition of correct spellings. He interpreted the result to mean that the college students retained the incorrect spellings and that the incorrect spellings interfered with efforts to retrieve the correct one. Given the assumption that people retain whatever answer they advance to a problem, the issue becomes to determine what answers they will advance. Three factors that we hypothesize influence the answers that preschoolers advance on addition problems are preexisting associations from the counting string, the sum of the numbers being added, and the frequency of exposure to the problems.

Counting-String Associations

Even before learning to add, most children know the counting string, at least as high as 10 (Pollio & Reinhardt, 1970; Pollio & Whitacre, 1970). Examination of the particular addition errors that children made in our experiments suggested that this prior knowledge of the counting string influences the formation of distributions of associations for adding. As shown in Fig. 9.7A, on all ties (problems on which the first and second number are the same, such as \( 3 + 3 \), and ascending-series problems (items on which the second number is larger than the first one, such as \( 2 + 4 \)), the most frequent error in Siegler and Robinson’s data and in the data of the replication and extension experiment was for children to state the number one greater than the second number. That is, they would say that \( 3 + 3 = 4 \) and that \( 3 + 5 = 6 \). Fig. 9.7B shows an identical pattern in the overt-strategies-prohibited experiment. Here too, the answer one greater than the second number was the most frequent error on all 10 ties and ascending-series problems. Our interpretation was that such problems triggered associations with children’s knowledge of the counting string and that the children simply advanced the next number in the series (e.g., \( 2 + 4 = 5 \)). The phenomenon is reminiscent of Winkelman and Schmidt’s (1974) finding that college students are slow to reject propositions such as \( 5 + 3 = 15 \)—that is, problems on which a different numerical operation provides an interfering association. It is also consistent with Miller and Gelman’s (in press) finding that counting string relations are the main determinant of 5-year-olds’ judgments of similarity among numbers.

The specific reason that counting-string associations influenced responses on ascending-series and tie problems but not on descending-series problems was open to speculation. The simplest account—that the more similar the problem to the counting string, the greater the likelihood of the association’s being triggered—fit the data only crudely. Problems with discrepant addends, such as \( 2 + 5 \), and tie problems, such as \( 3 + 3 \), elicited counting-string errors as often as problems that more closely paralleled the counting string, such as \( 3 + 4 \) and \( 4 + 5 \).

A somewhat less straightforward explanation was in considerably better accord with the data. The last addend in an addition problem may always activate its immediate successor as a potential answer. However, other knowledge that preschoolers have, namely that answers to addition problems should be at least as great as the larger addend, may prevent them from stating counting-string associates as answers on descending-series problems. On \( 4 + 1 \), for example, children would not say 2 as an answer because it did not exceed 4. This explanation was
consistent with the pattern of problems in which the counting-string associations did and did not influence answers, and also with three other pieces of evidence. First, in the addition experiments reported in this chapter, children rarely advanced answers smaller than the larger addend. They did so on only 5% of trials. Second, when we presented eight 4- and 5-year-olds a single digit (4, 5, or 6) and asked them to name the first number they thought of, all eight responded with the
The Sum of the Numbers

A second potential influence on the distribution of associations was the answers yielded by children's efforts to count their fingers or the objects in their mental images. Presumably, the more number of objects people need to count, the greater the probability that they will err somewhere in the counting process. Gelman and Gallistel (1978) provided evidence that 3-, 4-, and 5-year-olds do err more frequently in counting large than small sets. In the addition context, learning of responses advanced after incorrect counting of the objects in the representation would lead to problems with higher sums having flatter distributions of associative strengths than problems with smaller sums.

Frequency of Exposure to Problems

A third factor that could influence the formation of the distribution of associations is frequency of exposure to each problem. Parents, preschool teachers, and older children might present preschoolers with some problems more often than others. Greater opportunity to learn particular problems, rather than any inherent characteristics of the learners or the problems, might account for those problems having relatively peaked distributions of associations.

To test this view, Sue Hamman and I asked 30 adults, all parents of 2- to 4-year-olds who attended a local preschool, to teach their children about addition as they might at home. The parents were instructed:

We know that parents sometimes teach their children arithmetic problems even before the children enter elementary school. We are interested in learning how parents do this, since they often seem to be quite successful. Please give your child some addition problems as you might at home, so that we can learn how parents go about teaching.

The experimenter ended the session once the parent had presented 10 problems or once 5 minutes had passed, whichever came first.

The frequency with which parents presented various problems is shown in Table 9.5. The most frequently posed problems were ones that children answered quite accurately in the previous studies. The frequency with which parents presented the immediate successor of the number—that is, with the counting-string associate. Here, as on the addition problems, the absence of a larger prior number allowed the counting-string association to manifest itself. Third, Brush (1978) and Cooper, Starkey, Blevins, Goth, and Leitner (1978) found that 3- to 5-year-olds know that adding objects to a collection increases its number. How children acquire such knowledge, and how it combines with their other knowledge to influence addition performance, are unknown, but it seems at least plausible that counting-string associations always operate, but are manifested in children's answers only if the answers produced are greater than either addend.

The data indicate the number of parents (out of 30) who presented the given problem at least once.

sent the 25 problems generated by the combinations of augend (1 to 5) + addend (1 to 5) correlated \( r = .69 \) with percentage of errors in the Siegler and Robinson data, \( F(24) = 21.37, p < .01 \).

The parental input data also suggested why certain problems were easier than might have been expected on other grounds. Illustratively, in Siegler and Robinson and in the replication and extension experiment, children were more often correct on each ”+1” problem (e.g., 4 + 1) than on the inverse ”1+” problem (e.g., 1 + 4). Groen and Parkman (1975) and Svenson (1975) have reported similar findings on solution-time measures with first through third graders. Yet both the min model and the sum-squared model predict that the two types of problems should be of identical difficulty, and 1 + 4, unlike 4 + 1, has helpful counting-string associations working for it. Parental input may be a crucial factor in explaining this apparent anomaly. As shown in Table 9.5, parents presented ”+1” problems five times as often as they presented ”1+” items. Ties, which similarly have been found to be easier than the sizes of their minimum numbers or squared sums would suggest, were also presented relatively often. Thus, the frequency with which children encounter a problem, as well as the sizes of sums and associations from counting, seemed likely to be among the factors influencing the development of the distribution of associations. 12

Another interpretation of the parental input data is that the parents had accurate intuitions about which problems were easy to learn, and that these intuitions, rather than differential exposure, were responsible for the correlation between problem presentation rates and children's performance. We are currently testing this interpretation. If ”+1” problems are indeed easier to learn than ”1+” problems, then children who have not yet learned either of them should more quickly master the ”+1” items. With roughly one-fourth of the data from the experiment collected, this does not seem to be the case; the two types of problems seem equally difficult to learn. If the pattern continues, it would render unlikely the view that accurate parental intuitions about learnability created the correlation between parental input and problem difficulty.
To test the adequacy of this account of development, we quantified each of the hypothesized predictor variables and examined how well they together accounted for the Fig. 9.2A distribution of associations. The acquisition model was expressed as a regression equation with three independent variables. One predictor was associations from the counting string. All ties and ascending-series problems on which counting-string associations yielded a correct answer (e.g., 1 + 3) were assigned a value of two, all ties and ascending-series problems on which such associations yielded an incorrect answer (e.g., 2 + 3) were assigned a value of zero, and all descending-series problems (e.g., 3 + 2) were assigned a neutral value of one. A second predictor was the likelihood that children would err in counting the objects in their elaborated representations. This variable was defined operationally as the sum of the numbers in the problem, because the more objects children had to represent, and the more objects they had to count, the more likely they would be to err. The third predictor was the frequency of exposure to each problem. The Table 9.5 data on frequency of parental presentation of each problem was the operational measure of exposure to that problem.

The regression analysis supported the preceding analysis of development in several ways. The three predictor variables accounted for 85% of the variance in the percentage of errors on the 25 problems in Fig. 9.2A. Each of the predictors added significant amounts of variance to that explained by the other two. The sum variable was the first to enter the equation, accounting for 68% of the variance, the counting-associations variable was the next to enter, adding 10%, and the parental-input variable also added significant variance, 7%. These results were consistent with the view that preexisting associations from the counting string, likelihood of mistakes in counting objects in elaborated representations, and frequency of exposure to different problems contribute to the development of associations between problems and answers.

How might the process develop? Here, our account is largely speculative. The first issue is how children acquire each of the four strategies. Retrieval is as basic and as generally applicable a strategy as any that people possess. It almost certainly is present well before the preschool period. Retrieval seems inherently to entail standards akin to confidence criteria and search lengths. Illustratively, Mervis and Canada (1982) observed reliable “refusal responses” by 1-year-olds. That is, when infants were asked by their mothers to identify a given object from an array of objects in front of them, they consistently refused to choose any of them when none fit into the appropriate category. The refusal response did not arise when the named object was available. For the 1-year-olds to defy the social demand implicit in being asked for an object by their parents, they would need to know that each available object did not exceed their confidence criterion for belonging to the category. The fact that children at times said “there aren’t any” indicated that they also possessed a “stop rule” akin to a search length for governing the number of retrieval efforts they would make.

The way in which the other three strategies develop is at least as much a matter of speculation. The counting-fingers strategy is probably acquired through imitation of, and direct instruction by, other children and adults. Informal observation and discussion with parents indicates that such instruction is quite common. Whether the fingers and counting strategies develop independently of the counting-fingers approach, or whether they develop as attempts to execute or modify it, is unknown at present.

Another issue about the development of the process concerns how its three phases are assembled into an overall problem-solving procedure. Retrieval is so rapid, so generally applicable, and, ordinarily, so nearly effortless, that attempting it first may be the “default option” for a wide range of cognitive activity. Until children form an elaborated representation, they have nothing to count; thus, problem elaboration naturally precedes counting. In sum, assembling the process in the order retrieval first, problem elaboration second, and counting third does not seem to require great insight.

Might the process exist in a more rudimentary form before assuming the shape depicted in Fig. 9.2B? It seems plausible that at an early point in development, children might use only the retrieval strategy. This point would need to be quite early, however; Starkey and Gelman (1982) observed that 3-year-olds sometimes solved addition problems by putting up fingers (or putting out other objects) and counting them. Again, the existence of such a simple process at an early point in development is only speculation at present. Studying children younger than those we have examined to date may allow more solidly grounded statements about how the process develops.

A COMPUTER SIMULATION OF THE DEVELOPMENT OF THE DISTRIBUTION OF ASSOCIATIONS

A number of critics of information-processing approaches, among them Neisser (1976) and Beitin (1981), have argued that computer simulations have not, and probably cannot, account for development. They noted that most existing simulations of development generate performance but do not undergo transitions from one state to the next. Transition mechanisms are postulated at a verbal level, but not incorporated into the simulations themselves. In this section, we discuss a computer simulation of addition that incorporates learning and performance mechanisms into a single model and that proceeds from producing relatively poor performance to producing the sophisticated performance typical of 4- and 5-year-olds.

An Outline of the Simulation

The simulation can be described in terms of the beginning state of its representation and its process and in terms of six features of its operation. At the outset, the representation includes only two types of knowledge. One is the understanding that numbers as a general class are appropriate answers to addition problems. We
depicted this information as a set of minimal associations (associative strength = .01) between each problem under consideration and each possible answer (each whole number between 1 and 12). Second, when an ascending or tie problem is presented, the association between that problem and the answer one higher than the second number is momentarily strengthened, much as in semantic priming. For ease of description, and because this momentary association is identical to an enduring association in its effects on the probabilities of retrieving an answer and stating it, we have grouped together the two types of associations throughout the remainder of this discussion.

Now consider the initial state of the process. Biology has given it the ability to retrieve information from memory. Direct instruction and modeling have taught it to put up fingers (or their equivalent) and count them. Thus, the process at the beginning of the simulation resembles that shown in Fig. 9.2B.

This initial representation and process are insufficient to produce the performance of 4- and 5-year-olds. For example, when we presented to the simulation in its initial state a set of 10,000 problems, 400 each of the 25 problems in Siegler and Robinson, it stated a retrieved answer on only 7% of the trials. Percentage of errors and percentage of overt strategy use on each problem correlated r = .01. However, the initial representation and process do provide a base from which learning can occur. Six aspects of the acquisition process seem critical:

1. The simulation is presented the 25 problems in accord with their relative frequency in the parental input study described earlier.13
2. Before each problem, the simulation generates a confidence criterion and a search length. Both are selected by a random process, with the confidence criterion varying from .05 to .99 and the search length from 1 to 3.
3. The probability of retrieving an answer is proportional to its associative strength. A retrieved answer is stated if its associative strength exceeds the current confidence criterion. Retrieval attempts continue until either the associative strength of a retrieved answer exceeds the confidence criterion or the number of searches matches the allowed search length.
4. If the number of retrieval efforts reaches the allowed search length, and no answer has been stated, the program generates an elaborated representation of the number of objects in the augend and addend. In the program’s present implementation, this elaboration always corresponds to the fingers approach, in which the presentation, once generated, does not fade. Once the representation is generated, the model temporarily (for the duration of the trial) adds .05 to the associative strength of the answer corresponding to the number of objects represented. It then retrieves an answer, and states it if its associative strength exceeds the confidence criterion.
5. If this last retrieval effort is unsuccessful, the model counts the objects in the elaborated representation. On each count, there is a fixed probability of skipping over the object being counted and a fixed probability of counting it twice.
6. Every time the system advances an answer, the association between that answer and the problem increases. The increment is twice as great for correct answers, which presumably are reinforced, as for incorrect answers, which presumably are not.

The Simulation’s Behavior

The simulation runs in two phases: a learning phase and a test phase. The learning phase is designed to resemble children’s experience with addition prior to the time at which they enter the experiment. The test phase is intended to resemble behavior in the experimental setting, given children’s prior experience.

The learning phase includes 2000 trials, an average of 80 for each of the 25 problems (range = 60 to 158 trials per problem). During this phase, children develop more or less peaked distributions of associations on each problem. In accord with the previously discussed regression equations, three variables shape the learning process: associations from the counting string, frequency of presentation of each problem, and the sum of the numbers. To highlight the contribution of each of these variables, we compare pairs of problems that differ on that variable but whose status is the same on the other two variables.

First consider 1 + 4 and 2 + 3, problems that have identical sums and frequencies of presentation, but one with a helpful and one with an interfering association from counting. As shown in Fig. 9.8A, the item that has the helpful association, 1 + 4, rapidly builds a peak at the answer 5. The association between the item with the interfering association, 2 + 3, and the answer 5 starts from a lower point and grows more slowly. At the end of the learning phase, after 2000 trials, the answer 5 has 77% of the total associative strength for 1 + 4 versus 46% for 2 + 3. The peak for 1 + 4 is also higher in absolute terms: .67 versus .37. The greater percentage of total associative strength at 5 for 1 + 4 means that the simulation will retrieve 5 more often on this problem. The higher absolute peak for 1 + 4 means that when the simulation retrieves 5, it will state it more often.

Fig. 9.8B illustrates the developmental course for two problems that have identical sums and that lack specific counting-string associations, but that differ in frequency of presentation. The problem 4 + 1 is presented on 5.4% of trials, whereas the problem 3 + 2 is presented on 3.7%. The presentation rate has a

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13In the relatively small sample of parental input data that we had available at the time that this chapter was written, parents had never presented some problems (Table 9.5). This seemed unlikely to reflect the real-world input. Therefore, we added a constant to each percentage in the input data. This ensured that all problems would be encountered on at least 2% of trials, while also ensuring that the simulation most frequently encountered the most frequently presented problems.
marked effect on how high the peak rises as well as some effect on how peaked the distribution is. After 2000 trials, the absolute associative strength of the peak for $4 + 1$ was .80, whereas for $3 + 2$ the associative strength was .51. The percentages of associative strength that were located in the peak were 76% and 69%, respectively. These differences indicate that $4 + 1$ would be retrieved somewhat more often than $3 + 2$, and would be stated on a considerably higher percentage of those trials on which it was retrieved.

Finally, as shown in Fig. 9.8C, the sum exerted an effect even when the frequency of presentation and the type of counting-string association was constant. The problems $3 + 4$ and $4 + 5$ are identical in frequency of presentation and in having an interfering counting association. However, they differ in their sums. The peak of the item with the lower sum rises somewhat more rapidly and at the end of 2000 learning trials is higher (.39 versus .32) than that for the item with the higher sum. The distribution is also somewhat more peaked, with the peak of $3 + 4$ having 50% of the total associative strength and the peak of $4 + 5$ having 43%. Thus, all three variables influence the percentage of trials on which the correct answer is retrieved and the likelihood that it will be stated once it is retrieved.

After the simulation completes the learning phase, it proceeds to the test phase. Whereas the learning phase was intended to model children's experience prior to the experiment, the test phase was intended to parallel their experience in the experiment. The test phase differed from the learning phase in two respects. First, to parallel the empirical experiments that we conducted, all problems were presented equally often in the test phase. Second, because each child who participated in the empirical experiments received only two exposures to each problem, thereby providing very little opportunity to learn, we turned off the learning mechanism that added associative strength to each answer that was stated. The
goal was to model a large number of children, each having a brief experimental session, rather than a single child having a very long session.

The computer program's behavior in the test phase resembled that of children much more closely than had its behavior prior to the learning phase. It generated three of the four strategies: retrieval, fingers, and counting fingers (recall that the counting strategy has not yet been implemented in the simulation). The relative accuracies and the relative solution times of the three modeled strategies were identical to those of the children. The simulation's error patterns also were like the children's; the simulation's most frequent error on all 10 ascending and tie problems was the answer one greater than the second addend.

The simulation's performance also resembled that of the children in which problems elicited the greatest percentage of errors, which took the longest to answer, and which elicited the highest percentage of overt strategies. As shown in Table 9.6, all of the correlations of greatest interest between the simulation's behavior and that of the children exceeded $r = .80$. Moreover, the intrasimulation correlations among percentage of errors, percentage of overt strategy use, and mean solution times on each problem all exceeded $r = .90.$

The results of the simulation also suggested two hypotheses about the developmental sequence of addition skills. First, in examining the correlations between the model's and the children's percentages of overt strategies and errors, we noticed that the correlations between the two sets of errors started higher and grew more rapidly than did the correlations between the percentages of overt strategy use (Table 9.6). The reason lay in the dominating effect of the counting-string associations at the outset of the learning phase. These counting-string associations produce relative numbers of correct answers on the 25 problems much like those that will be present after the learning phase. The five problems with helpful associations remain relatively easy, and the 10 with interfering associations remain difficult. The effect on overt strategy use is different, however. The five problems with helpful associations continue to elicit relatively frequent use of the retrieval strategy. However, the 10 problems with interfering associations, which early elicit relatively frequent use of retrieval, are later among the problems that least often elicit use of retrieval. The reason is that before the learning phase, they are among the 15 problems whose distribution of associations have any peak, but after the learning phase, they are the only problems whose associative strength is divided among two peaks, one at the counting-string association and one at the correct answer. Thus, one testable hypothesis that emerges from the simulation is that the first age at which children's errors correlate significantly with the errors of 4- and 5-year-olds will be

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14 The model's solution times were operationally defined as the number of searches that it made before finding an answer on each problem.
earlier than the first age at which their pattern of overt strategy use correlates with that of the older children. We plan to test this hypothesis by presenting the problems to 34-year-olds and seeing which correlation emerges earlier.

A second, related prediction that follows from the computer model is that early in development, counting-string associations should be an especially powerful predictor of performance. At the outset of the computer simulation's learning phase, neither errors made in counting nor frequency of problem presentation has much effect on the distribution of associations. Their effects depend on children's having experience with the problems. Counting associations, however, exercise a strong effect from the outset. Thus, the percentage of responses in accord with counting-string associations should be greater early in development than later. Again, experiments with very young children can test these predictions.

In sum, a computer simulation that takes into account relative frequency of problem presentation, associations from the counting string, and the likelihood of errors in executing overt strategies can produce performance much like that of children. The simulation both learns and performs. At the outset, its performance is not very accurate, and is unlike that of 4- and 5-year-olds in many ways. After having an opportunity to learn, its performance is much more childlike. The simulation demonstrates that children could acquire their distributions of addition associations through the three hypothesized mechanisms, and that if they did, their performance would be much like what we observed.

DEVELOPMENT BEYOND THE PRESCHOOL PERIOD

Although we have not yet simulated development of addition skills beyond the level displayed by 4- and 5-year-olds, the simulation suggests ways in which the knowledge of older children and adults might develop. Ashcraft (1982), Groen and Parkman (1972), and Svenson (1975) have all noted two characteristics of 7- to 9-year-olds' performance on simple addition problems. The children make few errors, and their solution times are proportional to the size of the minimum addend. The present research complements these findings on the min strategy in at least four ways. It indicates the chronological period during which children learn the strategy, the range of problems on which they might apply it, the mechanisms by which they might decide when to use it, and how it could be integrated with the process that younger children use.

Our research provided almost no evidence of preschoolers' using the min strategy. In particular, on trials in which they counted aloud, preschoolers started counting from 1 rather than from the larger addend on more than 99% of trials. Thus, most children must acquire the min strategy at age 6 or 7—that is, in first or second grade.

9. STRATEGY CHOICES IN ADDITION AND SUBTRACTION

Our finding that even 4-year-olds accurately retrieve answers to many problems makes it unlikely that children use the min strategy on every problem once they do acquire it. Why would children use a reconstructive process when they could accurately retrieve the answer? Instead, children who use the min strategy probably use it only on relatively difficult problems—that is, on problems for which they possess relatively flat distributions of associations.

Older children could arrive at the min strategy through the same mechanisms by which 4- and 5-year-olds arrive at other overt strategies. They would use it when they could not in the allotted search length retrieve an answer whose associative strength exceeded the confidence criterion. As their distribution of associations for a problem grow more peaked, they would use the min strategy less and less frequently. This hypothesis is consistent with Ashcraft's (1982) finding that the minimum number ceases to be the best predictor of children's solution times after age 9.

Only two modifications would be needed to integrate the min strategy with the Fig. 9.2B process. First, the elaborated representation would include the number of objects indicated by the smaller addend, rather than the number indicated by the sum (Step 11). Second, counting would proceed from the number one greater than the larger addend rather than from "1" (Step 15). Otherwise, the process could be the same as that used by 4- and 5-year-olds. Such a process, like its predecessor, would have the desirable properties of yielding answers to all problems, of leading to use of the relatively effortless retrieval process on those problems in which it could yield consistently correct performance, and of eventually yielding a state in which retrieval would always be used on these simple addition problems. It would have the added advantage of reducing the number of objects that needed to be represented, thus reducing the time needed to solve problems and the likelihood of counting errors.

AN OVERVIEW OF THE MODEL

The present model of strategy choice in addition and subtraction is sufficiently complex that there may be a danger of losing the forest for the trees. Figure 9.9 provides an overview of the model that strips away much of the complexity. The distribution of associations, the key to the operation of the model, is at the center. Performance is produced by a three-phase process operating on this distribution. The three phases are retrieval, elaboration of the representation, and counting. Development of the distribution of associations is influenced by three factors: preexisting associations from the counting string, frequency of exposure to the problems, and the sum of the two addends. These three factors determine the peakedness of the distribution of associations, which in turn determines the percentage of overt strategy use, the percentage of errors on retrieval trials, and the mean solution times on retrieval trials.
Fig. 9.1. Model of strategy choice in addition.

9. STRATEGY CHOICES IN ADDITION AND SUBTRACTION

The model of strategy choice in addition is diagrammed in Fig. 9.1. The model is based on the idea that children use a variety of strategies to solve addition problems, and that these strategies are influenced by the nature of the problem and the child's knowledge of the problem space. The model includes four stages: preexisting associations from counting, frequency of exposure to each problem, calculation of associative strengths, and finally, the distribution of associative strengths. Each stage is dependent on the previous one, and the model operates on the assumption that children use a variety of strategies to solve addition problems, and that these strategies are influenced by the nature of the problem and the child's knowledge of the problem space.

The distribution of associations model provides a framework for considering strategy choices in domains as diverse as reading, spelling, and multiplication. The particular associations will vary among domains, but the organization of the process may be quite similar. In each case, the process could include three sequential phases. The first phase would be retrieval of an answer from a distribution of associations. The second phase, contingent on the failure of the first to yield a stable answer, would involve elaboration of the problem representation. The particulars of this elaborative process would vary with the task domain. The third phase, again contingent on the answer's not yet having been stated, would involve an algorithmic process certain to yield an answer but slower than the other strategies. As in the second phase, the particular algorithmic process varies with the domain of application. In all domains, the distribution of associations on particular problems would determine when each strategy was most likely to be used, when errors would most frequently occur, and when problems would be answered most rapidly.

Table 9.7 indicates how the model might be applied to the reading of words, spelling, and multiplication. Children use at least three strategies to decode words. Sometimes they retrieve a word's pronunciation and state it with no intervening overt behavior. Other times they proceed phoneme by phoneme in an attempt to "sound out" the word. Yet other times, they ask a parent, teacher, or older child to identify the word. The prediction of the model is that words that elicit frequent errors and long solution times should also elicit frequent use of "sounding out" and seeking help from other people.

Spelling is another domain in which people may base strategy choices on their distribution of associations. As discussed at the outset of this chapter, people can use at least four strategies to spell. They can retrieve the letters in a word, they can form a mental image of how the word might look, they can write out several possible spellings, or they can look in a dictionary. These strategies closely parallel the ones used in addition and subtraction. Familiar words, short words, words with unambiguous sound–letter correspondences, and words that conform to typical orthographic patterns would be expected to have peaked distributions of associations. They, therefore, also would be associated with short solution times, frequent correct answers, and frequent use of retrieval.

Multidigit multiplication presents another domain in which people might use a similar strategy-choice procedure. For some problems, such as 20 × 20, many people can retrieve the correct answer. For others, people might elaborate the problem representations in ways that take advantage of specific aspects of the numbers. For example, if asked to multiply 54 × 11, they might decompose 11 into 10 and 1, multiply each by 54, and finally add 540 and 54 to obtain the product. On yet other problems, people typically follow the long multiplication algorithm. This algorithm is applicable to all problems but, like looking up words in the dictionary or counting one's fingers, is cumbersome and time-consuming.

### TABLE 9.7

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<th>Task</th>
<th>Retrieval</th>
<th>Elaboration of Representation</th>
<th>Solution Algorithm</th>
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<tr>
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<td>Try to use &quot;shortcut&quot;</td>
<td>Use long multiplication algorithm</td>
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<td>Retrieve word's pronunciation</td>
<td>Sound out pronunciation</td>
<td>Ask the teacher</td>
</tr>
<tr>
<td>Spelling</td>
<td>Retrieve word's spelling</td>
<td>Write out alternative spellings and try to recognize correct one</td>
<td>Look up spelling in dictionary</td>
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SOME FINAL COMMENTS ABOUT STRATEGY CHOICE

In this concluding section, we discuss three issues raised by the present research: the role of explicit metacognitive decisions in strategy choices, the relation of the distribution of associations model to previous rule-oriented models that we have formulated, and the way in which the distribution of association model may help reconcile our intuitions of commonalities in reasoning across domains with the persistent empirical data to the contrary.

The Role of Metacognitive Decisions in Strategy Choices

The distribution of associations model illustrates a way in which people could choose strategies without extensive knowledge about their cognitive capacities and the demands of particular tasks. This type of model has several advantages. An obvious one is that the model provides a basis for the empirical finding that metacognitive knowledge often correlates weakly with strategy choices. Because it is possible to arrive at useful strategies without any metacognitive knowledge, the two do not need to be highly correlated.

Closely related, the model renders understandable the finding that despite 1- to 3-year-olds' minimal metacognitive knowledge, they make sensible strategy choices in certain situations. Specifically, in domains in which they have extensive experience, such as memory for object locations and production and comprehension of language, young children often use reasonable strategies (Clark, 1983; DeLoache, this volume; Perlmuter, 1980). In these domains, even young children have extensive associative networks to guide their performance. These associations may render explicit metacognitive knowledge unnecessary.

This view should not be confused with the frequently expressed hypothesis that young children are associative animals whereas older ones are conceptual (e.g., Kendler & Kendler, 1962; Lange, 1978; White, 1965). In our model, the process that operates on the associations is both powerful and general; it certainly seems to qualify for the title "conceptual." Adults and older children may sometimes use more abstract processes for choosing strategies that would not depend on associations within particular domains (as may young children). However, the problem isomorph literature (e.g., Kotovsky, 1983) suggests caution in postulating such processes and especially in hypothesizing their widespread use. Even adults frequently do not transfer strategies that they have learned in the context of a particular problem to other problems with identical structural characteristics. Specific associations between problems and means for answering them may often dominate adults' as well as young children's strategy choices.

It is impossible to demonstrate that a particular process does not occur in a problem-solving procedure. It can be demonstrated, however, that such a process would be redundant. In the present context, children could explicitly judge a problem's difficulty and their own problem-solving capacity, but it is difficult to see what purpose such judgments would serve. The model in its present form arrives at adaptive strategies without the judgments. Even if we postulated that children explicitly judged problem difficulty, we would still need to account for how they made their judgments and why the correlations among overt strategy use, errors, and solution times would be greatest for percentages of errors and mean solution times on retrieval trials. We would also need to explain how the four strategies were generated and to account for their relative solution times. In short, it is unclear what utility explicit metacognitive judgments would have within this model, either for children or for the researcher trying to model their thought processes.

It might be argued that the model already incorporates a metacognitive judgment in a slightly disguised form. This is the judgment that the associative strength of a retrieved answer does or does not exceed the confidence criterion. Due to the diversity of uses of the term metacognitive, we are unsure whether this judgment is best viewed as fitting within the metacognitive category. Regardless of its placement, the ability to make such decisions seems to us one of the most basic cognitive qualities. Without it, how would the 1-year-olds in the Mervis and Canada (1982) experiment have known not to answer their mothers' request when none of the available responses was appropriate? More generally, how would children know when to ask questions? Whether we label such judgments "metacognitive" is immaterial to our claim that on simple arithmetic problems, children choose strategies by a procedure like that shown in Fig. 9.2, rather than on the basis of their knowledge of their cognitive capacities, available strategies, and task demands.

The Relation of the Distribution of Associations Model to Rule Models

In previous research, one of us has characterized children's knowledge in terms of rules. Associations between problems and specific answers played no explicit role. Why might rule-based models be appropriate in some contexts and associative models in others?

A major difference between the tasks for which we have postulated rule-based models and the tasks for which we have postulated associative models concerns people's prior knowledge. Before the experimental session, people would have had little, if any, experience with balance scale, liquid-quantity conservation, or
fullness problems. They would certainly be unlikely to have formed strong associations between particular answers—such as that the left side of the balance scale will go down—and particular items, such as three weights on the third peg on the left side versus four weights on the second peg on the right. In contrast, children have extensive experience with specific addition, subtraction, spelling, and reading items. Associations between problems and particular answers would directly influence performance on many items.

Even in domains usually thought of as rule governed, associations involving particular items seem to play a crucial role when people have extensive experience with the domain. Support for this position comes from research on language development. Children’s overgeneralizations of the English past-tense and pluralization rules have often been cited as evidence for the rule-governed nature of language use (e.g., Brown, 1973; Cazden, 1968). Yet language rules never attain the consistency of those observed on the balance scale, liquid-quantity conservation, and other unfamiliar tasks. At all times, children continue to produce some correct irregular past-tense and plural forms (MacWhinney, 1978; Moratsos, 1983). These correct irregular uses tend to occur on the most common verbs, those for which children would have the greatest opportunity to form distributions of associations with strong peaks at the correct irregular form. Thus, models that focus exclusively on rules may be useful primarily in situations in which people have little experience with specific items. Associations involving particular items must also be considered in domains in which people have had experience.

Domain Specificity and Domain Generality

In the past decade, many cognitive developmentalists have been torn between their intuitions that commonalities in reasoning across domains do exist and the evidence that children reason in different ways on different tasks. Articles by Flavell (1982), Fischer (1980), Case (1978), and many others attest to this conflict. Each investigator has proposed means for reconciling the discrepancy. For example, Flavell suggested that commonalities may exist for some individuals, in some task domains, and at some points in the developmental process. The present research suggests an additional possibility. A common process, such as that depicted in Fig. 9.2B, may be utilized across numerous domains. Because the distributions of associations are distinct for each domain, indeed for each problem within each domain, this common process will not yield parallel behavior on different tasks. A child might form very peaked distributions for arithmetic problems, leading to good performance, and relatively flat ones on spelling items, leading to poor performance. The child’s behavior would not be parallel, but the process used to produce the behavior would be. Behavior comes about through processes’ operating on specific content knowledge. Only by considering the latter as well as the former are we likely to detect many of the unities in children’s thinking.

9. STRATEGY CHOICES IN ADDITION AND SUBTRACTION

ACKNOWLEDGMENTS

This research was supported by Grant #HD 16578 from the National Institute of Child Health and Human Development and by a grant from the Spencer Foundation. Useful comments and suggestions of William Chase, David Klahr, Brian MacWhinney, and Catherine Sophian on earlier versions of the manuscript are gratefully acknowledged.

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9. STRATEGY CHOICES IN ADDITION AND SUBTRACTION


Stemberg, R. J., & Ketron, J. L. Selection and implementation of strategies in reasoning by analogy. *Journal of Educational Psychology*, 1982, 74, 399–413.


**APPENDIX A: ZERO-ORDER CORRELATIONS FOR ALL PREDICTORS USED IN REGRESSION ANALYSES OF PERCENTAGE OF ERRORS AND MEAN SOLUTION TIME ON EACH PROBLEM**

**Siegler and Robinson Experiment**

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*All F statistics have 24 degrees of freedom.

**Replication and Extension Experiment—Group 1**

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*All F statistics have 24 degrees of freedom.

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*All F statistics have 24 degrees of freedom.

Replication and Extension Experiment—Group 3

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<th>Correlation</th>
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### Replication and Extension Experiment—Group 3 (Continued)

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*aAll F statistics have 24 degrees of freedom.

### Subtraction Experiment

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*aAll F statistics have 24 degrees of freedom.