A synesthetic walk on the mental number line: The size effect

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Abstract

Are small and large numbers represented similarly or differently on the mental number line? The size effect was used to argue that numbers are represented differently. However, recently it has been argued that the size effect is due to the comparison task and is not derived from the mental number line per se. Namely, it is due to the way that the mental number line is mapped onto the task-relevant output component. Here synesthesia was used to disentangle these two alternatives. In two naming experiments a digit-color synesthete showed that the congruity effect was modulated by number size. These results support the existence of a mental number line with a vaguer numerical representation as numbers increase in size. In addition, the results show that in digit-color synesthesia, colors can evoke numerical representation automatically.

Keywords: Mental number line; Size effect; Synesthesia; Bi-directionality
1. Introduction

A dominant metaphor for numerical representation is the mental number line (Restle, 1970). However, the exact representation of numbers on this mental line has been the subject of a longstanding debate. Some researchers proposed that numbers are represented in a vaguer\(^1\) way as numbers increase (e.g., logarithmic curve, Dehaene & Changeux, 1993; or linear curve with a scalar variability, Gallistel & Gelman, 1992). Others argued that representation on the number line is analog to the magnitude it represents (Zorzi & Butterworth, 1999). Recently Verguts, Fias, and Stevens (2005) challenged the idea of a vaguer representation for larger numbers (at least for single digits), and offered an elegant model based on place coding, linear scaling and fixed variability, which leads to similar representation for small and large numbers.

Two effects are attributed to the representation on a mental number line: (1) the numerical distance effect; a longer reaction time (RT) as the numerical distance between two digits decreases. For example, the decision whether 1 is larger than 2 is slower than the decision whether 1 is larger than 8. (2) The size effect; the comparison time of a pair of numbers when equated for numerical distance is faster for small numbers (e.g., 1–2), than for large numbers (e.g., 7–8) (Moyer & Landauer, 1967). Thus, the size effect is a strong indication for a vaguer representation for larger numbers.

Verguts et al. (2005) showed recently that the size effect may be task specific, that is, it appears with magnitude comparisons but not with other tasks that also require access to the mental number line, such as parity or naming tasks. These results led Verguts et al. to propose that the observance of the size effect under magnitude comparison stems from mappings of the number line to the task-relevant output component and not from the number line per se. This argument has been supported by a neuronal network model. In this model the weights of the connections between the mental number line and the output units for each task are task-dependent. Namely, under naming and parity tasks, the weights are equal for all the numbers. However, in the comparison task the weights produce a compressed scale. That is, there are different weights for small numbers and for large numbers. In another study, Verguts and Van Opstal (2005) showed dissociation between the distance effect and the size effect. Under a same-different task there was a distance effect whereas the size effect was absent. Such a result strengthens Verguts et al.’s (2005) proposal that the size effect does not originate from the mental number line. It is important to note that Verguts et al.’s model accounts only for symbolic notation, for non-symbolic input it predicts that the size effect should be observed due to increased variability, which appears only in later representation. Indeed, behavioral and neurophysiological studies with non-human primates showed that with non-symbolic input, a size effect is observed even when no comparison task is involved (e.g., Cantlon & Brannon, 2006; Nieder & Miller, 2003).

\(^1\) We used the term “vague” to define the argument for overlapped representation of numbers as their size increase.
Such an assumption implicitly assumes that the representation for symbolic and non-symbolic notation is different, and thus, that numerical representation is not amodal. Hence, it is highly important to further examine this model using symbolic stimuli.

**Tzelgov and Ganor-Stern (2004)** proposed that mental representations are best probed when their processing is automatic and not part of the requirement of the task that is intentionally performed. Under such conditions, the emerging picture is relatively uncontaminated by intentional strategies. Accordingly, in order to show that the distance effect and the size effect reflect basic features of the mental number line, the size congruity effect (SiCE) is a suitable paradigm. In this Stroop-like task, two digits are presented and participants are asked to decide which digit is physically larger while ignoring the numerical values of the digits. Commonly, participants cannot ignore the numerical values, which interfere with the physical judgments. That is, participants usually respond slower to incongruent pairs (e.g., 2 4) than to congruent pairs (e.g., 2 4). It has been shown that SiCE decreases as the irrelevant distance decreases (**Cohen Kadosh & Henik, 2006a; Henik & Tzelgov, 1982; Schwarz & Ischebeck, 2003; Tzelgov, Yehene, Kotler, & Alon, 2000**). Tzelgov and Guata (submitted for publication) recently found that SiCE decreased when the average location of the digit pair, on the number line, increased. Namely, SiCE was larger for the pair (1–2) than for the pair (8–9). These findings support the assumption of a vaguer mental number line as number size increases. In particular, according to the “Coalescence Model” (**Schwarz & Ischebeck, 2003**) the drift rate of information required for a decision is slowed down as accumulation of information from the irrelevant dimension becomes faster. The irrelevant dimension is processed faster, the larger the (irrelevant) numerical distance between the numbers compared. Hence, SiCE should decrease with the increase of pair location on the number line. Tzelgov and Guata’s (submitted for publication) findings were obtained when numerical magnitude of the numbers was not part of the task requirements. However, it could be argued that the requirement to compare physical sizes of digits triggered the comparison of numerical values, as provided by the experimental task (**Tzelgov & Ganor-Stern, 2004**). This in turn might activate the comparison output components (**Verguts et al., 2005**). Therefore, in order to support the argument that the size effect reflects a fundamental feature of the mental number line, it is important to show the size effect under conditions of automatic processing with no triggering of a comparison process. A naming task is ideal for this since it does not involve comparison, but results in access to the mental number line (**Brysbaert, 1995**). In the current study, we used a special naming task and synesthesia.

People with synesthesia experience the world in a different way than non-synesthetes. For example, digit-color synesthetes will see certain achromatic digits (i.e., inducer) in color (i.e., concurrent) (**Hubbard & Ramachandran, 2005; Rich & Mattingley, 2002; Robertson & Sagiv, 2004**).

Recently, it has been shown that the concurrent (e.g., color), can trigger the inducer (digit) (**Cohen Kadosh & Henik, 2006c; Cohen Kadosh et al., 2005; Knoch, Gianotti, Mohr, & Brugger, 2005**), thus, leading to the suggestion that synesthesia is
bi-directional (from digit to color, and vice versa), and not only unidirectional as was argued earlier (Mills, Boteler, & Oliver, 1999). However, the access to colors from digits is much more salient than when triggering digits from color (Cohen Kadosh et al., 2005). Previous research employed a variety of Stroop-like tasks to produce elevated RTs and errors when the inducer digit was colored in an incongruent concurrent color (Cohen Kadosh & Henik, 2006b; Dixon, Smilek, & Merikle, 2004; Mattingley, Rich, Yelland, & Bradshaw, 2001). The current study was carried out on a digit-color synesthete who showed bi-directionality in a previous report (Cohen Kadosh & Henik, 2006c).

In Experiment 1, the synesthete was asked to name a presented digit while ignoring the ink in which the digit was presented. We used small digits (i.e., 1 or 2) and large digits (i.e., 7 or 8), as in Verguts and Van Opstal’s (2005) study, colored in a congruent, or in an incongruent manner (Fig. 1a). In Experiment 2, the same stimuli as in Experiment 1 were employed, but the instructions were to name the digit that corresponded to the presented color while ignoring the presented digit (Fig. 1b).

If, on the number line, adjacent large numbers (e.g., 7–8) overlap more than adjacent small numbers (e.g., 1–2) (Dehaene & Changeux, 1993; Gallistel & Gelman, 1992), the congruity effect should be modulated by the size of the number. In contrast, if the representation for small and large numbers is similar (Verguts et al., 2005), we should observe an identical congruity effect in both Experiments 1 and 2, regardless of number magnitude.

Fig. 1. An example for an incongruent trial from Experiment 1 (a) and Experiment 2 (b). In the rectangle on the left is the correct verbal response (i.e., “ONE” in Experiment 1, “TWO” in Experiment 2). In the cloud shape on the right is the association triggered by the color green, which according to IS represents the number 2.
2. Experiment 1

2.1. Methods

2.1.1. Participants
IS is a 27-year-old, right-handed, male student, who experiences colors that are projected on stimuli such as digits, alphabetical letters, and days of the week. He did not experience any other type of synesthesia including visuo-spatial number forms (Sagiv, Simner, Collins, Butterworth, & Ward, 2005). We documented his strong bi-directional synesthesia in a previous work (Cohen Kadosh & Henik, 2006c).

2.1.2. Stimuli
A stimulus display consisted of a colored digit on a yellow background (yellow was not associated with any number by IS). The stimuli subtended a vertical/horizontal visual angle of 1.8°/0.8°–1.3°. A digit could appear in the color it triggered (congruent condition), or in the color of an adjacent digit (1 in the color of 2 (green), 2 in the color of 1 (gray), 7 in the color of 8 (brown), 8 in the color of 7 (dark green)) (incongruent condition). We used the digits 7–8, rather than 8–9, since Verguts and Van Opstal (2005) suggested that the similarity between 8 and 9 might cause a greater interference, not due to the greater overlap as numbers increase in size, but due to the physical similarity.

The entire experiment contained 2 sessions with 128 trials each. A block of 8 practice trials preceded each session and was similar to the experimental block. The study was approved by the local ethics committee.

2.1.3. Procedure
The participant’s task was to name the presented digit while ignoring its color. He was asked to name the digit as quickly as possible but to avoid errors. Verbal responses were monitored using a microphone connected to a voice onset relay, and the experimenter recorded the participant’s response.

Each trial began with a turquoise fixation point, presented for 500 ms. Five hundred milliseconds after the fixation point disappeared, a colored digit appeared and remained in view until the participant initiated a vocal response. A new trial began after the experimenter documented the verbal response.

2.1.4. Design
The variables manipulated were congruity (incongruent or congruent), and number size (small or large). Thus, we had a 2 × 2 factorial design. We measured RT and accuracy as a function of this manipulation.

2.2. Results

2.2.1. Phenomenological level
IS reported that the colors evoked the corresponding numbers in his mind’s eye.
2.2.2. Behavioral level

RTs were calculated for correct trials only, and were subjected to a two-way analysis of variance.

The main effects for congruity $[F(1,249) = 28.83, p < 0.001]$ and number size $[F(1,249) = 5.93, p < 0.05]$ were significant. Crucially, the interaction between congruity and number size was significant $[F(1,249) = 4.47, p < 0.05]$. Simple comparisons revealed that the congruity effect of 53 ms under small size was significant $[F(1,249) = 5.31, p < 0.05]$ and the congruity effect of 124 ms under large size was highly significant $[F(1,249) = 27.90, p < 0.001, \text{Fig. 2}].$

2.3. Discussion

The congruity effect was modulated by number size; the larger the digits the greater the congruity effect. The current experiment involved a naming task and no magnitude comparison. Hence, these results challenge Verguts et al.’s (2005), and Verguts and Van Opstal’s (2005) argument that the size effect is task specific and is not contingent upon the characteristics of the mental number line. Note that the current results cannot be explained by the relative speed account (Schwarz & Ischebeck, 2003) since the congruent condition yielded a similar RT.

3. Experiment 2

3.1. Methods

All methodological details were identical to those of Experiment 1. However, IS was asked to name the digit he associated with the color of a presented digit, while ignoring the presented digit.
3.2. Results

The main effect of congruity was significant \( F(1,250) = 17.22, p < 0.001 \). Importantly, the interaction between congruity and number size was also significant \( F(1,250) = 8.37, p < 0.005 \), Fig. 3. Simple comparisons revealed that the congruity effect of 100 ms under small size was significant \( F(1,250) = 24.60, p < 0.001 \). In contrast, the congruity effect of 17 ms under large size was not significant \( F < 1 \). In addition, the processing time for the congruent condition differed for small and large numbers \( F(1,250) = 5.2, p < 0.05 \).

4. General discussion

The current results show a modulation of the congruity effect by number size under a naming task. Namely, in Experiment 1 we observed a larger congruity effect as number size increased. In Experiment 2, we observed a reduced congruity effect as number size decreased. When IS is presented with a colored digit, the magnitudes of the presented digit’s value and the value of the font’s color are activated on the mental number line. In both experiments one of the activated magnitudes is the basis of the required response: in Experiment 1 it is the magnitude of the presented digit, and in Experiment 2 it is the magnitude that corresponds to the color. Based on the synesthete’s phenomenology, it makes sense to assume that the pathway from the presented digit to its name is stronger (or at least more easily activated) than the pathway from the color to the number corresponding to it. That means that in both experiments there will be a bias to respond with the name of the number presented. Thus, it seems that the numerical value of the digit was available to IS before the value resulting from the color of the digit was accessed. When the task was to name
the presented digit (Experiment 1), this bias speeded up responding, which in turn left less time for the interference of the irrelevant dimension. This was equally true for large and small numbers. How does it help to explain the larger interference in the case of larger digits? It seems that the larger interference for larger numbers found in Experiment 1 was due to “reverse priming”; the priming from the numerical value corresponding to the relevant dimension (the printed digit) to the numerical value corresponding to the irrelevant dimension (the color of the font), which in case of the incongruent condition, increased inhibition. According to this notion (Glaser & Glaser, 1989; La Heij, 1988; Neumann, 1986), there is a spread of activation from the target (the presented digit) to the distractor (the number that is activated by the color), emerging due to the color of the font. Models of semantic priming (e.g., Neely, 1977) assume that such priming is larger, the closer the target and distractor. Thus, the greater the overlap between two adjacent numbers, the larger the priming (and the interference) from the presented digit to the magnitude corresponding to the color. Therefore, the interference in Experiment 1 is mainly due to reverse priming, which in turn led to interference by the early available information from the target. In this sense it may be stated that this interference is semantic, that is, caused by a spread of activation within the semantic network.

In Experiment 2 when IS had to name the digit corresponding to the color of the font, the source of the interference was different. According to our view, the size effect reflects a basic feature of the mental number line. Thus, smaller numbers are more discriminable, which in turn leads to faster access of their numerical value. Hence, according to the relative speed account (Schwarz & Ischebeck, 2003), an irrelevant 2 will have more chance to interfere with the processing of the numbers adjacent to it than an irrelevant 8. This leads to an increased interference for small numbers and to total absence of interference for large numbers. According to this interpretation, the source of the interference in Experiment 2 is pre-semantic, namely, it happens before access to the semantic meaning.

The current results support the idea that numbers are represented on the mental number line with a greater overlap between the numbers as their size increases (Dehaene & Changeux, 1993; Gallistel & Gelman, 1992; Zorzi & Butterworth, 1999). These results are in contrast to Verguts et al.’s (2005) model, which assumes equal intervals between the numbers along the mental number line. Thus, it seems that the results of the current experiments together with Tzelgov and Guata’s SiCE results (submitted for publication), each using a different paradigm, support the assumption of an overlapped representation of numbers as their size increase.

In the current research, we used a single-case study. Although this impacts the generalization of the current findings to the entire synesthetic population, it still allowed us to examine two competing theories of numerical cognition. In addition, in Sagiv et al.’s (2005) study, synesthetes and non-synesthetes show similarity in their numerical representation. This finding suggests that the way IS represents numbers is similar to the way non-synesthete participants do.

While our findings support the claim that the size effect reflects a basic feature of numerical representation, they are non-informative concerning the ongoing arguments of whether the number line reflects ordinality or cardinality of the number line.
(Sagiv et al., 2005). Namely, our findings do not support any specific model of the mental number line. While Dehaene and Changeux’s (1993) logarithmic model and Zorzi and Butterworth’s (1999) analog magnitude model imply that the representation on the number line reflects the cardinal characteristics of numbers, Gallistel and Gelman’s (1992) linear curve with a scalar variability model is consistent with both views on this issue.

The current study extends previous studies that showed that synesthesia might be bi-directional (Cohen Kadosh & Henik, 2006c; Cohen Kadosh et al., 2005; Knoch et al., 2005). The observance of an interaction between congruity and number size in Experiment 1, points out that color evokes numerical representation that is very similar to the presented numbers (i.e., size effect). If there was no bi-directionality, incongruent colors would not activate the corresponding numbers. In this case, the congruity effect would be due to simple mismatch, and would be comparable regardless of number size.

In Experiment 1, the interfering digit was not presented on the screen. Such interference is considered to stem from a highly automatic processing, since the activation of numerical representation induced by the color is not part of the task, is not beneficial, and needs no triggering (Tzelgov & Ganor-Stern, 2004). Hence, the current results are free from intentionally applied strategies. Such results indicate that in the current case, the bi-directionality is highly automatic.

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