SPACE, TIME AND NUMBER IN THE BRAIN

SEARCHING FOR THE FOUNDATIONS OF MATHEMATICAL THOUGHT
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Improving Low-Income Children’s Number Sense

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Summary

This article describes how a theoretical analysis and empirical findings regarding number sense led to the development of an educational intervention that produces large and rapid increases in low-income children’s mathematical knowledge. Roughly an hour of playing a simple numerical board game based on the mental number line construct led to substantial gains in their knowledge of numerical magnitudes, counting, numeral identification, number line estimation, and arithmetic. The gains remained present two months after the last game-playing session. Both physical features of the game board and the way in which children interact with it proved important in the size of the gains. Reasons why such a brief intervention produces such substantial learning were discussed.

Throughout the developed world, the mathematical knowledge of children from low-income families lags behind that of children from wealthier families [1]. Even before children enter school, the differences are seen on a wide range of foundational tasks: counting from one, counting up or down from numbers other than one, recognizing written numerals, comparing numerical magnitudes, adding, and subtracting [2,3].

These early differences in numerical knowledge have lasting consequences. Kindergartners’ mathematical knowledge is strongly predictive of their mathematics achievement in third grade, fifth grade, eighth grade, and even high school [4–6]. The long-term predictive relations of early mathematical knowledge are unusually strong, more than twice as strong as the relations in reading, attention control, and emotional self-regulation [4]. Moreover, absolute differences in
mathematical knowledge between children from richer and poorer backgrounds, already substantial in kindergarten, steadily widen over the course of schooling [7].

These differences in the mathematical knowledge of children from richer and poorer backgrounds reflect differences in environmental support for learning mathematics. Middle-income parents more frequently engage in mathematical activities with their children than do low-income parents [3,8], and children whose parents present them with more mathematical activities generally have greater mathematical knowledge [9]. A study of home, preschool and daycare environments indicated that most children from working-class backgrounds received mathematical input in 0 of the 180 segments observed [10].

One result of the minimal mathematical input they receive is that children from low-income families often enter school with poorly developed number sense. There is widespread agreement that acquiring number sense is an important part of mathematical development and an important goal of mathematics instruction (e.g., [11–13]). However, reaching this goal is harder than it sounds. One reason that improving number sense is difficult is that there is little agreement on what number sense is. This lack of an accepted definition has contributed to difficulties in knowing how to study number sense and how to help children acquire it.

A review of the literature [14] suggested one promising definition of number sense: “a process of translating between alternative quantitative representations.” The translations can be between spatial and numerical representations (e.g., “About how many feet wide is this room?”), temporal and tactile representations (e.g., “tap your finger once every 10 s”), luminance and pressure (e.g., “the brighter this light, the harder you should press on this pad”), and so on. This definition suggests three key questions for understanding number sense: How can we best think about number sense? How can we measure children’s ability to approximate numerical magnitudes? and How can we help improve children’s number sense, including their approximation of numerical magnitudes?

With regard to the first question, the core of number sense seems to us to be the presence of a linearly increasing mental number line. A wide range of theories of numerical cognition propose that knowledge of whole numbers is organized around a mental number line, in which number symbols (e.g., “7”) are connected to nonverbal representations of quantity in an ordered, horizontally oriented array. The nonverbal representations of quantity appear to be largely spatial (e.g., [15]), though other sensory modalities also seem to be included in the representation [16]. Both behavioral and neural data support the mental number line construct. One body of evidence comes from studies of the SNARC Effect (spatial–numerical associations of response codes), the tendency of people in cultures with left-to-right orthographies to respond faster on the left to smaller numbers and on the right to larger numbers. For example people more quickly answer the question, “Which is bigger, seven or four?” when seven is chosen with a right side key press than with a left side key press [17]. A second source of evidence comes from brain-damaged patients with left-side neglect who displace upward (rightward on the number line) their bisections of numerical ranges (e.g., they estimate that the midpoint of the range 11–19 is 17), just as they do with physical lines [18]. A third set of evidence comes from brain imaging studies. The horizontal, intraparietal sulcus (HIPS), a brain area believed to be central to the mental number line, shows greater activation during comparison of numbers close in magnitude than during comparison of numbers further apart, presumably because finer magnitude discriminations require greater activation of relevant brain areas [19,20]. Fourth and especially important, the precision of
children’s approximation of numerical magnitudes and their mental number line representations has been found to be quite highly correlated with their performance in arithmetic, memory for numbers, and overall mathematics achievement test scores [21,22]. Thus, our strategy for improving number sense in children from low-income backgrounds was to help them generate a mental number line that precisely and accurately related symbolically expressed numbers to nonverbal numerical representations.

With regard to the second question, estimation tasks, especially number line estimation tasks, have several advantages for measuring and investigating number sense. As shown in Fig. 21.1, the number line estimation task involves asking children to translate between numerical and spatial representations, for example, asking them to estimate the location of 74 on a number line with 0 at one end and 100 at the other. This task has several desirable characteristics. Number line estimation can be used with any real number—large or small, positive or negative, whole number or fraction. The task transparently reflects the ratio characteristics of the number system. Just as 80 is twice as large as 40, the estimated location of 80 should be twice as far from 0 as the estimated location of 40. It is non-routine; neither parents nor teachers typically instruct children in how to do number line estimation, so children’s sense of the magnitudes of the numbers is reflected in their estimates.

Estimated values on the number line should increase linearly with a slope of 1 with the size of the number being estimated. Thus, the distance on the physical number line between 0 and the estimated location of 20 should be the same as the distance between the estimates of 20 and 40, 40 and 60, 60 and 80, and 80 and 100. The number line task allows several measures of the quality of children’s estimates, including the percent absolute error (the absolute distance between the number that was presented and the number corresponding to the child’s estimate on the number line), the slope of the best fitting linear function relating the number presented to the child’s estimate on the number line, and whether the best-fitting function relating the number presented to the child’s estimates is linear, logarithmic, or exponential.

Although the linear relation between the number presented and the estimate on the number line might seem obvious, children do not understand it for a surprisingly long time. Instead, children seem to progress through a rough three-step progression in which they first lack knowledge of even the ordinal properties of symbolically expressed numbers, then know the numbers’ order but do not relate them in a linearly increasing fashion, and then represent the relation between numbers and their magnitudes as increasing linearly, as in the equation \( y = x \). With regard to the first period, many three- and four-year-olds who count flawlessly from one to 10 do not even know the rank order of the numbers. For example, their percent correct on magnitude comparison tasks is close to chance, and they show knowledge of only the smallest
numbers (e.g., 1 and 2) on other tasks [23]. Somewhat older children know the rank order of the numbers but still fail to understand that equal differences between two pairs of numbers mean that the magnitudes of the two pairs of numbers must be equally discrepant. Instead, these children use a logarithmic representation of numerical magnitudes, in which estimates of magnitudes at the low end of the scale are farther apart than estimates at the high end for any given difference between numbers. Thus, as shown in Fig. 21.2, most kindergartners, and about half of first graders, think that on a 0–100 number line, the magnitudes of 12 and 20 are much more discrepant than the magnitudes of 72 and 80. Not until second grade do the number line estimates of most children indicate understanding that these differences are equal, and not until fourth grade do most children show similar knowledge on 0–1000 number lines [21,24].

The number line estimation findings are far from isolated phenomena. Similar patterns of developmental changes have been found with other types of estimation, including numerosity estimation (“Here’s a beaker with one dot and here’s a beaker with 1000 dots. Put about N dots in this empty glass on the screen by holding down the mouse”) and measurement estimation (“Here’s a line 1 zip long and here’s a line 1000 zips long; draw a line N zips long.”) Most children either generate logarithmic estimation patterns on all three estimation tasks or generate linear estimation patterns on all three [21]. Perhaps most striking, the linearity of number line estimates of children in all grades through fourth grade correlate substantially with the children’s mathematics achievement test scores [21,25,26].

Given that children can count from one to 10 at least a year before they show knowledge of the magnitudes of knowledge in this range, counting is clearly insufficient for generating accurate numerical magnitude representations. This raises the question of what other experiences might contribute. One common activity that might help children generate linear representations is playing linear, number board games—that is, board games with linearly arranged, consecutively numbered, equal-size spaces (e.g., *Chutes and Ladders.*) These board games provide multiple cues to numbers’ magnitudes. The greater the number in a square, the greater: (a) the distance that the child has moved the token; (b) the number of discrete moves of the token the child has made; (c) the number of number names the child has spoken; (d) the number of number names the child has heard; and (e) the amount of time since the game began. Thus, children playing the game have the opportunity to relate the number
in each square to the time, distance, and number of manual and vocal actions required to reach that number. Stated differently, these temporal, visual–spatial, kinesthetic, and vocal cues provide a broadly based, multi-modal foundation for a linear representation of numerical magnitudes. This theoretical analysis, together with the data on developmental and individual differences in estimation, suggested that providing children with numerical board game experience might improve their number sense and their performance on a wide range of numerical tasks. For example, given the linear relation between numbers and the spatial dimension of the board game, experience playing the game should make it possible for children to generate linearly increasing estimates on the number line task.

**DOES PLAYING NUMERICAL BOARD GAMES IMPROVE CHILDREN’S NUMBER SENSE?**

To test whether playing number board games promotes number sense, we randomly assigned 36 four-year-olds to play either a number board game or a color board game (Fig. 21.3) [27]. A little more than half of the children were African Americans attending Head Start or childcare centers that served impoverished populations.

At the beginning of each session, children in the number board condition were told that on each turn, they would spin a spinner that would point to “1” or “2”, that they should move their token that number of spaces, and that the first player to reach the end would...
win. Children in the color board condition were told that on each turn, they would spin a spinner that could point to different colors, that they should move their token to the nearest square with the same color as the one to which the spinner pointed, and that the first player to reach the end would win. The experimenter also told children to say the numbers (colors) on the spaces through which they moved. Thus, children in the number board group who were on a 3 and spun a 2 would say, “4, 5” as they moved their token. Children in the color board group who were on green and spun a “blue” would say “purple, blue.” If a child erred or could not name the numbers or colors, the experimenter correctly named them and then had the child repeat the names while moving the token.

The preschoolers played the number game or the color game about 20 times over four 15- to 20-min sessions within a two-week period; each game lasted about 3 min. At the beginning of Session 1 and at the end of Session 4, children were presented the 0–10 number line estimation task as a pretest and post-test.

Playing the number board game led to dramatic improvements in the low-income preschoolers’ number line estimates. On the pretest, the best-fitting linear function accounted for 15% of the variance in individual children’s estimates; on the post-test, it accounted for 61%. In contrast, for children in the color board game condition, the best fitting linear function accounted for 18% of the variance on both pretest and post-test. Thus, playing the number board game for four 15- to 20-min sessions over a two-week period produced substantial improvements in low-income children’s number line estimation.

**GENERATIONAL OF LEARNING ACROSS TASKS AND TIME**

We [28] tested the generality of the benefits of playing the number board game, both in terms of the range of numerical knowledge that children acquire and in terms of the stability of learning over time. Effects of playing the number and color board games on understanding of the numbers one to 10 were compared on four tasks: number line estimation, magnitude comparison (“Which is bigger: N or M”), numeral identification (“Read the number on this card”), and counting (“Count from 1 to 10”). Playing the number board game was expected to produce gains on the magnitude comparison task for the same reason as on the number line task—improved understanding of numerical magnitudes. Playing this game also was expected to improve counting and numeral identification, because it provides practice and feedback on those skills too. Performance on the four tasks was assessed not only on a pretest and immediate post-test but also on a follow-up nine weeks after the final game playing session. The participants were four- and five-year-olds from Head Start centers, slightly more than half of them African American.

As in the previous study, accuracy of number line estimation increased from pretest to post-test among children who played the number board game. Gains remained present on the nine-week follow-up. In contrast, there was no change in the accuracy of estimates of children who played the color board game. The same pattern was evident on all four numerical tasks (Fig. 21.4). In all cases, preschoolers who played the number board game showed improvements that persisted over time, whereas peers who played the color board game showed neither immediate nor delayed improvements.
The linear numerical board game is not the only focused intervention aimed at improving the numerical magnitude understanding of preschoolers from low-income backgrounds that has yielded encouraging results. A software program known as “The Number Race” [29–31] has also yielded promising findings, in this case with French preschoolers from low-income backgrounds. The focus of this program is on improving number sense access, defined as linking nonverbal number sense to symbolic representations of numbers. The construct of number sense access is similar to the present construct of translations between alternative quantitative representations. The Number Race involved adaptive computer software designed to help children compare numerical magnitudes, link symbolic and non-symbolic representations of number, and increase understanding of and skill in arithmetic. The Number Race produced gains in numerical magnitude comparison performance and a cross-format matching task that measured understanding of numerical magnitudes, though not arithmetic skill.

**GAME PLAYING IN THE EVERYDAY ENVIRONMENT**

The results with numerical board games raised the question of what role in numerical development, if any, board games occupy in the everyday environment. To address this issue, we obtained self-reports about preschoolers’ experiences with board games, card games, and video games [28]. The self-reports were obtained from the preschoolers from...
low-income backgrounds who participated in the experiment described in the last section, and from age peers from middle-income backgrounds. We hypothesized that children from middle-income backgrounds would have greater experience with board games and that a child’s amount of experience playing board games would correlate positively with that child’s numerical knowledge.

The data were consistent with both hypotheses. Children from middle-income backgrounds reported twice as much experience with board games as children from low-income backgrounds. Interestingly, children from middle-income backgrounds reported less video game experience than their peers from low-income backgrounds. Within the low-income sample (the only group for which we had numerical proficiency data), the amount of board game experience correlated positively with all four measures of numerical knowledge. Whether preschoolers reported having played Chutes and Ladders, the commercial game that seems closest to the present board game, also correlated positively with their performance on three of the four numerical tasks. In contrast, the amount of experience with video games and with card games correlated with proficiency on only one of the four numerical tasks. Thus, both correlational and causal evidence point to a connection between playing numerical board games and acquiring numerical knowledge.

**WHICH FEATURES OF BOARD GAMES INFLUENCE LEARNING?**

Experiments designed to identify the critical features of the number board game have allowed tests of the theory that stimulated design of the game and have provided valuable data for future applications. Both physical features of the games and features of game-playing activity have proved important.

The linearity of the game board has proven to be one vital feature. The linear board was predicted to produce greater learning than a circular board, because the mapping between the linear physical board and the desired mental number line is simpler. The greater learning could arise either from a propensity to form a nonverbal mental number line being innate, and providing a framework to which numerical information could be linked, or from a mental number line having already started to form, due to experiences seeing people counting objects in a 1:1 number word:object fashion from left to right.

The prediction that playing the game with a linear board would produce greater learning than playing it with a circular board proved to be correct [32]. The same study demonstrated that playing the linear game, but not the circular one, leads to a higher percentage of correct answers to addition problems and to a higher percentage of errors that are close in magnitude to the correct answer. This finding was anticipated because brain areas activated in addition have been shown to be highly similar to areas activated on tasks that measure analog magnitude representations [33]. The greater impact of playing the game with the linear board on the number line, magnitude comparison, and addition tasks supports the view that playing the linear board game improves children’s number sense.

A feature of game-playing activity that has proven important is counting-on from the number where the token starts the turn (as opposed to counting from one). Because children do not automatically encode written numerals before third grade [34], requiring counting-on was hypothesized to be crucial for the children to encode the numbers in the squares,
which was believed crucial for them to learn the magnitudes of those numbers. The reason was that if children did not encode the symbolically expressed number (e.g., identifying the square with “8” as the “8”-square and the square with “4” as the “4”-square), they would have nothing with which to correlate the spatial, temporal, kinesthetic, and verbal cues. To the extent that identifying the number within each square was crucial to learning, and to the extent that children would not encode the number if they did not have to state the number as part of the counting-on procedure, learning would be reduced.

We tested this hypothesis by presenting kindergarten children with a $10 \times 10$ game board displaying the numbers $1–100$. As predicted, kindergartners who were required to count-on by saying the numbers in the squares (e.g., “36, 37”) learned much more about numerical magnitudes than children required to count “1, 2” in the same situation [35]. Both their number line estimation and their magnitude comparison improved. In addition, they were able to more accurately estimate the locations of numbers on the game board, thus demonstrating that the counting-on procedure led to better encoding of the numbers’ positions on the board, which was hypothesized to be essential for learning about numerical magnitudes through playing the board game.

**EFFECTS OF OTHER PRESCHOOL MATHEMATICS INTERVENTIONS**

Over the years, many large-scale interventions designed to improve the mathematical skills of low-income preschoolers have been implemented. Like the board game interventions emphasized in this chapter, these multifaceted interventions show that with proper support, children from low-income backgrounds can improve their mathematical knowledge considerably. Below we review three of the interventions that have received the most convincing empirical support: Number Worlds (e.g., [36]), Building Blocks (e.g., [8]), and Pre-K Mathematics (e.g., [3]).

Number Worlds (formerly known as Rightstart) focuses on teaching children the underlying concept of number before moving on to formal addition and subtraction. As such, the program ensures that children have a good conceptualization of addition, subtraction, and numerical magnitudes using real objects (e.g., four pegs) before introducing more symbolic representations (e.g., the numeral 4). The curriculum includes a wide range of numerical activities: songs about numbers, counting games, games involving money, board games somewhat similar to the one that we have used, and so on. Within the curriculum, children spend approximately 20 min per day involved in mathematical activities.

Number Worlds has proven very successful in increasing children’s numerical knowledge; after 40 sessions, 87% of low-income kindergarteners passed a test of basic numerical skills that was passed by only 25% of peers who did not receive the curriculum [37,38]. The children remained more advanced at the end of first grade (despite only receiving the intervention during kindergarten) and their first grade teachers rated them as having better number sense than other children from similar backgrounds.

The second curriculum, Building Blocks, features small- and whole-group activities, computer games and family activities designed for home use [8,39]. The curriculum focuses on varied aspects of mathematics including number, patterns, and geometry. Children spend at least 1 h focused on math each week for 26 weeks.
In a large randomized trial, 35 lower- and middle-class preschool classrooms were chosen to present the Building Blocks curriculum, another math-focused curriculum, or the curriculum used the previous year. Children who received the Building Blocks curriculum improved their math skills more than children who received the previous year’s curriculum or the comparison curriculum.

A third broad preschool mathematics curriculum, Pre-K Mathematics, also combines school-based and home-based activities [3,40]. Children participate in small-group activities for 20 min twice a week throughout the school year. Parents are provided with home activities that link to the small-group activities in the school and include hands-on manipulatives along with instructions on how to perform each activity. Children from impoverished backgrounds who participated in the Pre-K Mathematics intervention showed equivalent mathematical knowledge on the post-test to middle-income children who did not participate [3].

While they are effective, these interventions are very costly, in terms of both time and resources. In addition to the money that preschools have to spend to purchase the curriculum, there is also substantial cost in terms of teacher training. For Number Worlds, teachers who implemented the curriculum received assistance from the researchers twice a week throughout the school year. For Building Blocks, teachers received 34 h of group training and 16 h of in-class coaching. For Pre-K Mathematics, teachers participated in eight days of workshops over the course of the school year, along with on-site training at least once per month. There is good reason for this extensive teacher training: without substantial and prolonged guidance, teachers often modify the curricula in ways that make it less effective [41,42].

CONCLUSIONS

The high costs of large-scale curricula were part of what motivated us to apply theoretical understanding of cognitive development, in particular findings regarding the centrality of the mental number line to mathematical understanding, to devise a focused intervention. In addition to providing information about the effectiveness of the particular intervention, this approach yielded two other types of valuable information: knowledge useful for deepening theoretical understanding of numerical magnitude representations, and knowledge useful for improving large scale interventions aimed at helping preschoolers gain numerical understanding.

Consider some of the lessons regarding acquisition of numerical magnitude representations. Some of the main conclusions are that one source of acquisition of linear representations of numerical magnitudes is playing linear numerical board games, that improved numerical magnitude representations also lead to improved ability to learn answers to arithmetic problems, and that playing linear number board games produce greater learning of both numerical magnitudes and arithmetic than playing circular ones. The results also support the view that experience that allows children to correlate symbolically expressed numbers with redundant nonverbal cues to the magnitudes associated with those symbols is crucial to the process of learning mathematics.

The lessons learned from the focused interventions also can, and we believe should, be incorporated into large-scale curricula that are aimed at improving the mathematical understanding of preschoolers from low-income backgrounds. Including games like the present ones might improve the effectiveness of the large-scale curricula; including them might also make it possible for the large-scale curricula to produce gains more quickly.
The success of focused interventions such as the one emphasized in this chapter raises the question of why such brief interventions can produce such great improvements in the mathematical knowledge of preschoolers from low-income families. One likely reason is that many of these children have had few explicitly mathematical experiences prior to the interventions. Systematic observations of home and preschool environments of young children from low-income backgrounds indicate that neither environment typically provides many experiences where the children’s attention is directed to numbers [10,43]. For many children, the experience with numerical magnitudes that they receive even in the relatively brief interventions may constitute a substantial percentage of their experience connecting symbolically expressed numbers to their non-symbolic equivalents and relating symbolically expressed numbers to each other. Because of these children’s limited numerical experience, because early differences in mathematical understanding tend to persist throughout schooling, and because of the large, broad, and rapid effects of early interventions that are grounded in cognitive and developmental theories and data, increasing the number of preschoolers who receive such interventions seems a goal worth pursuing.

References


