CHILD DEVELOPMENT PERSPECTIVES

An Integrative Theory of Numerical Development

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ABSTRACT—***Understanding of numerical development is growing rapidly, but the volume and diversity of findings can make it difficult to perceive any coherence in the process. The integrative theory of numerical development posits that a coherent theme does exist—progressive broadening of the set of numbers whose magnitudes can be accurately represented—and that this theme unifies numerical development from infancy to adulthood. From this perspective, development of numerical representations involves four major acquisitions: (a) representing magnitudes of nonsymbolic numbers increasingly precisely, (b) linking nonsymbolic to symbolic numerical representations, (c) extending understanding to increasingly large whole numbers, and (d) extending understanding to all rational numbers. Thus, the mental number line expands rightward to encompass larger whole numbers, leftward to encompass negatives, and interstitially to include fractions and decimals.***

KEYWORDS—*numerical development; numerical magnitudes; mathematical development; fractions; negative numbers; number line*

Research on numerical development is expanding rapidly, with large literatures emerging on numerical development in infancy, childhood, and adolescence; development of subitizing, counting, estimation, and arithmetic; knowledge of whole numbers, fractions, decimals, and negatives; and nonsymbolic and symbolic representations. Researchers have also examined conceptual and procedural knowledge; underpinnings of numerical development in evolutionary processes, neural processes, cognitive processes, and emotional processes; longitudinal stability of individual differences; and numerical competence in normal and special populations. The list does not end there: Researchers have also looked at relations to numerical knowledge of variations in economic status, culture, language, and instruction; relations among numerical, spatial, and temporal knowledge; relations of numerical knowledge to more advanced mathematics; and relations of interventions that improve numerical knowledge to subsequent learning, to name a subset of areas within the field (see Table S1 in the online Supporting Information for references for each area). Discoveries in these areas attest to the health of the field of numerical development. However, the sheer number of discoveries and areas can make it difficult to perceive any coherence in the developmental process. Is there such coherence or is numerical development just one thing after another?

THE INTEGRATED THEORY OF NUMERICAL DEVELOPMENT

The integrated theory of numerical development proposes that the continuing growth of understanding of numerical magnitudes provides a unifying theme for numerical development. Within this perspective, numerical development is a process of broadening the set of numbers whose magnitudes, individually or in arithmetic combination, can be accurately represented. The theory identifies four main trends in numerical development: (a) representing increasingly precisely the magnitudes of numbers expressed nonsymbolically, (b) linking nonsymbolic to symbolic representations of numerical magnitudes, (c) extending the range of whole numbers whose magnitudes can be represented accurately, and (d) representing accurately the magnitudes of numbers other than whole numbers, particularly fractions, decimals, and negatives.

The integrative theory begins with the popular metaphor of the mental number line. However, it goes on to propose that this

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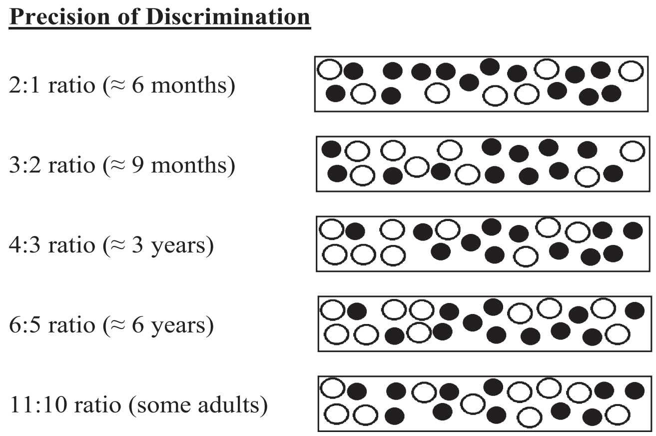

Figure 1. **The development of knowledge of *nonsymbolic* numerical magnitudes. The sets of black and white dots represent experimental stimuli whose numerosity can be discriminated reliably at the specified ages.**

mental number line is a dynamic, continually changing structure rather than a fixed, static one. Initially useful for organizing knowledge of nonsymbolic numbers and then of small, positive, symbolic whole numbers, the mental number line is progressively extended rightward to represent larger symbolic whole numbers, leftward to represent negative numbers, and interstitially to represent symbolic fractions and decimals (see Figures 1 and 2).

Both correlational and causal data support the integrated theory's emphasis on the importance of accurately representing numerical magnitudes. Preschoolers' success in identifying the more numerous of two dot collections predicts their math achievement as much as 2 years later, even after controlling for other intellectual variables (Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011). Accuracy of number line estimation correlates substantially with overall mathematics achievement from kindergarten through at least eighth grade (Booth & Siegler, 2006; Siegler & Booth, 2004; Siegler, Thompson, & Schneider, 2011). The accuracy of first graders' location of symbolic whole numbers on number lines predicts the accuracy of their fraction number line estimation and fraction arithmetic in seventh and eighth grades, even after controlling for IQ, working memory, and socioeconomic status (SES; Bailey, Siegler, & Geary, 2014). Moreover, manipulations that improve representations of whole number magnitudes improve subsequent learning of whole number arithmetic (Booth & Siegler, 2008; Siegler & Ramani, 2009), and manipulations aimed at improving fraction magnitude representations improve learning of fraction arithmetic (Fuchs et al., 2013, 2014).

These studies demonstrate the importance of numerical magnitude representations from early childhood through adolescence. The acquisition of knowledge of nonsymbolic numerical magnitudes actually begins even earlier, in infancy.

NONSYMBOLIC REPRESENTATIONS OF NUMERICAL MAGNITUDES

Long before children learn number words, they represent numerical magnitudes nonverbally. The mechanism by which people (and many other species) do so has been labeled the Approximate Number System (ANS; Halberda, Mazzocco, & Feigenson, 2008). From early in infancy, the ANS allows discrimination between sets of objects in which the ratio of the larger to the smaller set is sufficiently large, independent of the area and perimeter of the objects, their luminance, and other potentially confounding variables. Discriminability between sets is a function of the ratio of their number of items, as described by Weber's law. For example, discriminating between 8 and 12 objects and between 16 and 24 is equally difficult (De Smedt, Noël, Gilmore, & Ansari, 2013). A second magnitude representation mechanism, sometimes termed *object files*, also exists early in infancy and yields more precise discrimination between sets of 1–4 objects than the general ratio-based mechanism would produce (Feigenson, Dehaene, & Spelke, 2004). These patterns are sometimes described in terms of distance and magnitude effects; discrimination between set sizes is more accurate when the set sizes are more discrepant (distance effects) and involve fewer objects (magnitude effects).

The precision of nonsymbolic number discrimination increases considerably over the first few years. At 6 months, infants discriminate 2:1 ratios, but not until 9 months do they discriminate 3:2 ratios (Wood & Spelke, 2005). The improvement continues well beyond infancy; 3-year-olds consistently discriminate dot displays that differ by 4:3 ratios, 6-year-olds discriminate displays that differ by 6:5 ratios, and some adults discriminate displays that differ by 11:10 ratios (Halberda & Feigenson, 2008; Piazza et al., 2010).

Discrimination of nonsymbolic numerical magnitudes might seem an isolated skill of little importance, but individual differences in the skill at 6 months are related to mathematics achievement on standard symbolic mathematics tasks at 3 years, even after statistically controlling for IQ (Starr, Libertus, & Brannon, 2013). Moreover, individual differences at age 3 years are related to scores on standardized mathematics achievement tests concurrently and 2 years later (Mazzocco et al., 2011).

However, three literature reviews, two including meta-analyses, indicate that relations between ANS acuity and math achievement are weaker and less consistent than relations between representations of symbolic numerical magnitude and math achievement (Chen & Li, 2014; De Smedt et al., 2013; Fazio, Bailey, Thompson, & Siegler, 2014). Furthermore, symbolic numerical knowledge has been found to fully mediate the relation between nonsymbolic numerical knowledge and mathematics achievement in both 4-year-olds (VanMarle, Chu, Li, & Geary, 2014) and 6-year-olds (Göbel, Watson, Lervåg, & Hulme, 2014). Nonetheless, knowledge of nonsymbolic

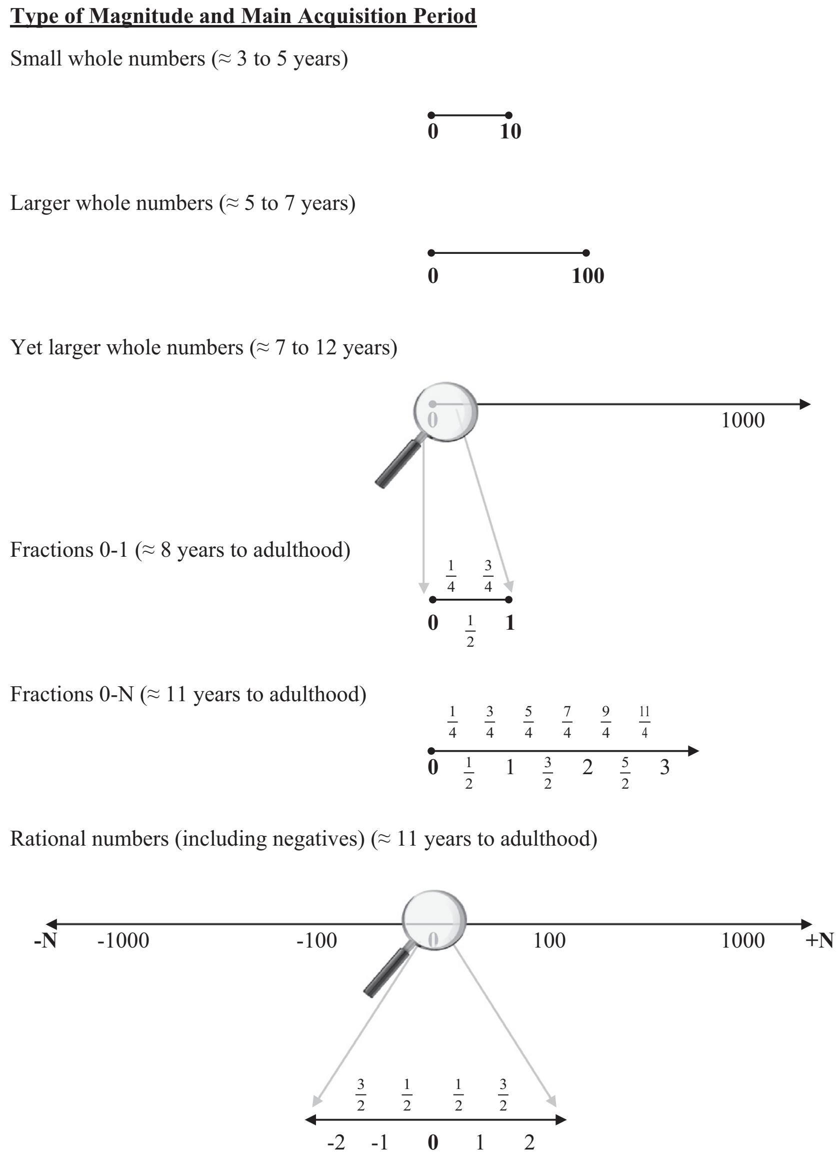

Figure 2. **The development of knowledge of *symbolic* numerical magnitudes. Knowledge of symbolic numerical magnitudes expands from small whole numbers to larger whole numbers to rational numbers, including common fractions, decimals, and negatives. The ages associated with the expansions indicate the period in which knowledge of each type of magnitude typically shows the greatest increases.**

magnitude seems to provide a foundation for understanding the referents of at least small symbolic numbers.

FROM NONSYMBOLIC TO SYMBOLIC REPRESENTATIONS OF NUMERICAL MAGNITUDES

Behavioral and neural data show several striking parallels between representations of nonsymbolic and symbolic magnitude. Behaviorally, the same distance and magnitude effects that are shown with nonsymbolic magnitudes are shown with symbolic ones, and the mathematical functions relating solution times to problem characteristics also are similar (Moyer & Landauer, 1967). At the neural level, not only do brain areas involved with representations of symbolic and nonsymbolic magnitude correspond closely, but habituation of either symbolic or nonsymbolic representations produces habituation of the other, as measured by functional magnetic resonance imaging (fMRI) activations (Piazza, Pinel, Le Bihan, & Dehaene, 2007).

Despite these similarities, the process of connecting symbolic to nonsymbolic numerical magnitude representations is surprisingly slow and piecemeal. On a task in which 3- and 4-year-olds were asked to give the experimenter *N* objects, some children who could count to 10 gave the correct number of objects for only the number 1; others only the numbers 1 and 2; others only the numbers 1, 2, and 3; and others only the numbers 1, 2, 3, and 4 (Le Corre, Van de Walle, Brannon, & Carey, 2006). Despite being able to count accurately sets of 5–10 objects, many of these children assigned the numbers 5–10 to sets of objects in ways uncorrelated with the set size. Not until age 4½ did most children respond to number words beyond 4 on the "Give X Task" in ways correlated with the sets' magnitudes. Other paradigms have yielded similar results (e.g., Schaeffer, Eggleston, & Scott, 1974), indicating that even with very small whole numbers, connecting symbolic numbers to their magnitudes develops slowly.

REPRESENTING AN INCREASING RANGE OF WHOLE NUMBER MAGNITUDES

Even after children know the relative magnitudes of the numbers 1–10, they continue to have limited knowledge of the magnitudes of larger numbers. The acquisition of knowledge of the magnitudes of two-, three-, and four-digit whole numbers, like the acquisition of knowledge of the magnitudes of single-digit ones, is slow and piecemeal. This is apparent in number line estimation. On this task, children are presented a series of lines with a constant pair of numbers at the two ends (e.g., 0 and 100) and asked to locate a series of other numbers on the line (one number per line). Afterward, alternative mathematical functions are fit to the estimates to establish the one that best describes the estimation pattern.

With symbolic as with nonsymbolic numbers, the psychological distance between numbers at the low end of the range is much larger than that between numbers of identical arithmetic distance at the high end of the range. Thus, when asked to locate symbolically expressed numbers on a 0–10 number line, 3- and 4-year-olds space their estimates of small numbers (e.g., 2 and 3) much farther apart than their estimates of large numbers (e.g., 7 and 8), whereas 5- and 6-year-olds space the two pairs of numbers equally (Bertelletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010). The younger children's estimates increase logarithmically with the sizes of the number being estimated, whereas the estimates of the older children increase linearly.

The developmental sequence repeats itself at older ages with larger numbers. Thus, in the 0–100 range, 5- and 6-year-olds generate logarithmically increasing patterns of estimates, whereas 7- and 8-year-olds' estimates increase linearly (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004). In the 0–1,000 range, 7- and 8-year-olds generate logarithmically increasing patterns of estimates, but 9- and 10-year-olds generate linearly increasing ones (Booth & Siegler, 2006; Siegler & Opfer, 2003). The same 7- and 8-year-olds who consistently produced linear estimation patterns on 0–100 number lines produced logarithmic patterns on 0–1,000 lines (Siegler & Opfer, 2003), reflecting the gradual extension of numerical magnitude knowledge to larger whole numbers.

Performance on number line estimation and other measures of numerical magnitude knowledge with symbolically expressed numbers is related strongly to other aspects of mathematical knowledge. Accuracy and linearity of number line estimation of symbolic whole number magnitudes for both the 0–100 and 0–1,000 ranges correlate strongly with arithmetic proficiency (Booth & Siegler, 2008; Ramani & Siegler, 2008) and overall mathematics achievement (Geary et al., 2007). The relation between symbolic numerical magnitude estimation and mathematics achievement remains even after statistically controlling plausible third variables including arithmetic, reading achievement, and IQ (Bailey et al., 2014; Booth & Siegler, 2006, 2008; Geary et al., 2007). Furthermore, having children play a number board game designed to improve representations of symbolic magnitude yields gains not only in their magnitude knowledge but also in other tasks such as learning novel arithmetic problems (Ramani & Siegler, 2008; Siegler & Ramani, 2009).

These findings suggest that arithmetic is far from the rote memorization process that it is often portrayed as being. The role of magnitude knowledge in arithmetic is evident on verification tasks, where both children and adults consistently take longer to reject incorrect answers that are close in magnitude (e.g., $6 \times 8 = 46$) than incorrect answers that are distant in magnitude (e.g., $6 \times 8 = 26$; Ashcraft, 1982). This role of magnitude knowledge in arithmetic is also evident in spontaneous retrieval errors, which usually are close in magnitude to the correct answer (Lemaire & Siegler, 1995). Magnitude knowledge can lead to activation of plausible answers, detection of implausible answers, and substitution of correct procedures for flawed ones that produce implausible answers.

FROM WHOLE NUMBERS TO RATIONAL NUMBERS

Developing knowledge of nonsymbolic rational numbers shows similarities to acquisition of nonsymbolic whole numbers. For instance, 6-month-olds discriminate 2:1 but not 3:2 ratios, just as they do with whole numbers. Thus, when habituated to a 2:1 ratio of blue and yellow dots, they dishabituate when shown a 4:1 ratio but not when shown a 3:1 ratio (McCrink & Wynn, 2007). Moreover, just as specific neurons are tuned to respond maximally to specific whole numbers, specific neurons are tuned to respond maximally to specific ratios (Jacob & Nieder, 2009).

Developing symbolically expressed whole numbers and fractions also shows several differences, the most obvious being that acquisition of knowledge of symbolic fractions begins much later and never reaches as high a level. Thus, even adults attending community college are only 70% accurate in comparing magnitudes of fractions, whereas they are almost 100% accurate in comparing corresponding magnitudes of whole numbers (DeWolf, Grounds, Bassok, & Holyoak, 2014).

Within the integrated theory of numerical development, acquiring knowledge of symbolic fractions requires learning that several invariant properties of whole numbers are not invariant properties of all numbers. Whole numbers have unique successors, can be represented by a single symbol, never decrease with multiplication, never increase with division, and so on. In contrast, none of these qualities are invariant for rational numbers. However, whole and rational numbers are alike in having magnitudes that can be represented on number lines. Thus, understanding rational numbers requires learning that many invariant properties of whole numbers are not true for rational numbers, and also learning the mapping between symbolically expressed rational numbers and the magnitudes they represent.

Consistent with this analysis, despite the obvious differences between understanding whole number and fraction magnitudes, the two show similarities. One similarity is that brain regions associated with fraction magnitude representations overlap with those associated with whole number magnitude representations (Ischebeck, Schocke, & Delazer, 2009). In addition, both show distance effects on magnitude comparison tasks: For fractions with unequal numerators and denominators, the closer the fraction magnitudes being compared, the longer the comparison takes (Meert, Grégoire, & Noël, 2009; Schneider & Siegler, 2010). Furthermore, with fractions as with whole numbers, individual differences in magnitude knowledge correlate highly with individual differences in arithmetic and overall math achievement (Bailey, Hoard, Nugent, & Geary, 2012; Siegler et al., 2011), even when reading achievement and executive functioning are controlled statistically (Siegler & Pyke, 2013). Moreover, longitudinal data show that 6-year-olds' knowledge of whole number magnitudes predicts 13-year-olds' knowledge of fraction magnitudes (Bailey et al., 2014), even after controlling for the common influence of IQ, working memory, and SES. Finally, the positive effects on subsequent arithmetic learning of training designed to increase knowledge of whole number magnitude extends to fractions: not only does training aimed at improving fraction magnitude knowledge also improve fraction arithmetic but also gains in fraction magnitude knowledge mediate improvements in fraction arithmetic learning (Fuchs et al., 2013, 2014).

Fewer studies have been conducted on representations of negative than positive numbers, and none of them appears to be with negative fractions. However, on the basis of the limited available data, magnitudes seem to play a similarly central role in representations of negative and positive numbers. Brain regions associated with representations of the magnitudes of negative numbers overlap considerably with those associated with representations of the magnitudes of positive numbers (Gullick, Wolford, & Temple, 2012). Developmental changes in brain activity associated with the two also show parallels; with negative as with positive numbers, activation of frontal areas on magnitude comparison tasks decreases from childhood to adulthood, whereas activation of parietal areas increases (Gullick & Wolford, 2013). Behavioral evidence indicates that as with positive numbers, both 10- to 12-year-olds and adults show distance effects with negatives on problems of magnitude comparison (Ganor-Stern, Pinhas, Kallai, & Tzelgov, 2010; Gullick & Wolford, 2013). Moreover, the size of distance effects with positive numbers is related to the size of distance effects with negative numbers, at least for 10-year-olds (Gullick & Wolford, 2013). Thus, although the research base is scanty, magnitude knowledge appears to play a similar role with negative and positive numbers.

CONCLUSIONS

A basic tenet of the integrated theory is that numerical development is largely a process of broadening the range and types of numbers whose magnitudes are well understood. The developmental process includes at least four trends: representing nonsymbolic numerical magnitudes increasingly precisely, linking nonsymbolic and symbolic representations of small whole numbers, extending the range of numbers whose magnitudes are accurately represented to larger whole numbers, and representing accurately the magnitudes of rational numbers, including fractions, decimals, percentages, and negatives. These trends begin at different ages, and the level of mastery reached by adulthood varies considerably among different types of numbers, but many commonalities, both neural and behavioral, are evident in the acquisition process. Individual differences in mastery of all types of numerical magnitudes are linked to individual differences in arithmetic proficiency and mathematics achievement, and experiences that improve magnitude representations also improve other numerical skills, such as arithmetic learning. Thus, accurate representations of numerical magnitude can be seen as the common core of numerical development.

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SUPPORTING INFORMATION

Additional supporting information may be found in the online version of this article:

Table S1. Some of the Numerous Areas of Numerical Development Research, With Illustrative References