Tian, J., & Siegler, R. S. (2018). Which type of rational numbers should students learn first? *Educational Psychology Review. 30*(2), 351-372. doi: 10.1007/s10648-017-9417-3.

Which Type of Rational Numbers Should Students Learn First?

Tian, Jing[1,2] • Siegler, Robert S.[1,2]

Abstract:
Many children and adults have difficulty gaining a comprehensive understanding of rational numbers. Although fractions are taught before decimals and percentages in many countries, including the USA, a number of researchers have argued that decimals are easier to learn than fractions and therefore teaching them first might mitigate children's difficulty with rational numbers in general. We evaluate this proposal by discussing evidence regarding whether decimals are in fact easier to understand than fractions and whether teaching decimals before fractions leads to superior learning. Our review indicates that decimals are not generally easier to understand than fractions, though they are easier on some tasks. Learners have similar difficulty in understanding fraction and decimal magnitudes, arithmetic, and density, as well as with converting from either notation to the other. There was too little research on knowledge of percentages to include them in the comparisons or to establish the ideal order of instruction of the three types of rational numbers. Although existing research is insufficient to determine the best sequence for teaching the three rational number formats, we recommend several types of research that could help in addressing the issue in the future.

Keywords:
Decimals; Fractions; Magnitude; Arithmetic; Rational numbers

Mathematics proficiency greatly influences success in school and beyond (Duncan et al. 2007; Koedel and Tyhurst 2012; Ritchie and Bates 2013), and knowledge of rational numbers is essential to that proficiency. For example, examination of nationally representative longitudinal data sets from the United States and the United Kingdom showed that fraction knowledge in 5th grade uniquely predicts algebra and general math

---

Correspondence can be sent to Jing Tian at jingtianpsy2012@gmail.com and to Robert S. Siegler at rss2169@tc.columbia.edu

1 Department of Human Development, Columbia University-Teachers College, Grace Dodge Hall, Fourth Floor, 525 West 120th St, Box 118, New York, NY 10027

2 Siegler Center for Innovative Learning and Advanced Technology Center, Beijing Normal University, Beijing, 100875 China

achievement in high school, even after adjusting for IQ, working memory, whole number arithmetic, and family background (Siegler et al. 2012). Several other studies show similar positive relations between earlier rational number knowledge and later mathematical proficiency over shorter time periods (Booth et al. 2014; Booth and Newton 2012; DeWolf et al. 2015a; Geary et al. 2012).

The importance of rational numbers extends beyond the school years. Poor understanding of rational numbers precludes later participation in many middle- and upper-income jobs (McCloskey 2007; Sformo 2008). In a survey of a nationally representative sample of U.S. workers from diverse job categories (including upper level white collar, lower level white collar, upper level blue collar, and lower level blue collar), 68% of participants reported using rational numbers at work (Handel 2016). Moreover, rational numbers are ubiquitous in daily life, such as adjusting recipes to the number of guests and calculating taxes and tips (Jordan et al. 2013; Lortie-Forgues et al. 2015).

Unfortunately, many people have little understanding of rational numbers. Consider data on fractions. On a National Assessment of Educational Progress (NAEP; Martin et al. 2007), 50% of 8th graders failed to order three fractions (2/7, 5/9, and 1/12) from least to greatest. On the 2004 NAEP, only 29% of eleventh graders correctly translated 0.029 to 29/1000 (Kloosterman 2010). The lack of understanding goes beyond individual fractions. On the 1978 NAEP, when asked to choose the closest number to 12/13+7/8 from among 1, 2, 19, 21, and "I don't know", only 24% of the 8th graders chose the correct answer "2" (Carpenter et al. 1980).

The poor performance on fraction tasks is not limited to standardized tests. In small group and one-on-one testing settings, many students and even mathematics teachers reveal limited knowledge of fractions (Behr et al. 1984; Hanson and Hogan 2000; Newton 2008; Siegler and Pyke 2013). For example, in Siegler et al. (2011), accuracy of fraction arithmetic problems with numerators and denominators of five or less was 32% among 6th graders and 60% among 8th graders. Similarly, in Ma (1999), only 43% of math teachers who were interviewed provided a correct answer for $1\frac{3}{4} \div \frac{1}{2}$, and only 4% generated a conceptually correct representation of the problem.

Many efforts have been made to improve instruction in rational numbers (e.g., Cramer et al. 2002; Fosnot and Dolk 2002; Lamon 2012; Siegler et al. 2010; Smith et al. 2005), but in the past 30 years, at least, students' knowledge has shown little progress. To cite one example, Lortie-Forgues et al. (2015) presented 8th graders the previously described problem 12/13 + 7/8 and found that the percent correct increased only from 24% in 1978 to 27% in 2014. Even less encouraging, accuracy among 17-year-olds on three NAEP items that involved multiplying a fraction by a whole number decreased by about 20 percentage points from 1978 to 2004 (Kloosterman 2010).

The importance of mastering rational numbers, the failure to do so by many students, and the limited progress that students have made through the years underscore the importance of developing more effective rational number instruction. One popular proposal is that decimals should be taught before fractions, on the logic that they are easier to understand (DeWolf et al. 2014, 2015b; Ganor-Stern 2013; Hurst and Cordes 2016; Iuculano and Butterworth 2011; Johnson 1956; Zhang et al. 2013). This suggestion differs from the current standard practice in the U.S. and many other countries of teaching fractions before decimals or percentages (Australian Curriculum and Assessment Reporting Authority [ACARA] 2014; Common Core State Standards Initiative [CCSSI] 2010; Department for Education 2013).

The purpose of this review is to discuss which rational number format students should learn first. We pursue this goal by evaluating evidence regarding the assumption that decimals are easier to understand than fractions, as well as by examining whether instructional approaches that present decimals and percentages before fractions result in superior learning. Where evidence is inadequate for addressing key issues, which is fairly often the case, we note the gaps and recommend specific types of research that would address them.

The review includes five parts. The first part presents an analysis of the rational number construct and motivations for considering alternative sequences for teaching rational number notations. The second part discusses the hypothesis that understanding decimals is easier than understanding fractions and therefore that teaching decimals before fractions might yield superior instructional outcomes. The third part reviews data on whether children's understanding of decimals is, in fact, superior to their understanding of fractions, as well as data on sources of difficulty in learning fractions and decimals. The fourth part reviews the limited existing data on understanding of percentages, which currently preclude analysis of whether percentages are more or less difficult to understand than decimals and fractions. Finally, in the fifth part, effects of an instructional intervention that taught both percentages and decimals before fractions are reviewed, and implications of these and other findings for instruction are discussed. Only positive rational numbers are discussed in this review, because the challenges learners face in understanding negatives and zero are quite different (Blair et al. 2012).

Why Consider Instructional Sequences for Teaching Rational Number Notations?

Rational numbers are a complex construct, in that they have multiple interpretations and can be expressed in multiple notations. One influential theory (Kieren 1976, 1980) distinguishes among five major interpretations of rational numbers: part-whole, ratio, operator, quotient, and measure. The idea that rational numbers are a multi-faceted construct has been further developed by several other researchers (see Behr et al. 1992 for a review). Moreover, rational numbers can be represented with three related but different notations: fractions, decimals, and percentages. Each notation can express the multiple interpretations of rational numbers, although people prefer particular notations for expressing particular interpretations (DeWolf et al. 2015b; Tian and Siegler, in preparation). Comprehensive knowledge of rational numbers requires understanding the multiple interpretations of rational numbers, skill at translating among the three notations, and knowledge of when each numerical notation is most convenient to use (Behr et al. 1983, 1992; Charalambous and Pitta-Pantazi 2007; Kieren 1976, 1980).

Some researchers have proposed that decimals should be taught before fractions, because decimals are easier to understand than fractions (DeWolf et al. 2014, 2015b; Hurst and Cordes 2016; Iuculano and Butterworth 2011; Johnson 1956; Zhang et al. 2013). If decimals are indeed easier to learn, then providing instruction on them first and then utilizing knowledge of them to teach about fractions (and perhaps percentages) may reduce children's overall difficulty in learning rational numbers. Whether decimals are indeed easier to learn than fractions is unclear, however, as is whether teaching them first improves learning of the other two rational number notations. In the following section, we discuss the reasoning behind the claim that decimals are easier to learn than other rational number notations, as well as reasons to question the claim.

Reasoning Behind the View that Decimals Are Easier Than Fractions

A number of researchers have proposed that decimals are easier to understand than fractions. One of their main arguments is that decimal notation is more similar to the already mastered whole number notation (DeWolf et al. 2014, 2015b; Hurst and Cordes 2016; Johnson 1956). For example, DeWolf et al. (2014) argued that, "the greater ease of comparing decimals than fractions, coupled with the overall similarity of decimal and integer comparisons, strongly suggests that the formal similarity of decimals and integers underlies the relative ease of processing the latter number types" (p. 81). Similarly, Johnson (1956) noted that decimals use the same place-value system as whole numbers, and thus that arithmetic with decimals is more straightforward than with fractions. He commented on the practice of teaching fraction arithmetic, instead of decimal arithmetic, immediately after children learnt whole number arithmetic:

> That is to say we have to have our children learn all of the four operations in addition, subtraction, multiplication and division not only in whole numbers where place value is the central theme of understanding but in common fractions where there is no place value causing the operation to be performed in an entirely different manner subject to rules altogether different when there is a simpler way that operates by the principle of place value (i.e., decimals) thus using and reinforcing a technique which is already known. (p. 202)

However, it is uncertain whether the greater similarity between decimal and whole number notations yields better understanding of decimals than fractions. Understanding the properties of rational numbers goes far beyond the ability to interpret the symbols. For example, even children who can interpret the symbols 0.2 and 0.3 may not know that there are an infinite number of numbers between 0.2 and 0.3, nor that multiplying 0.2 and 0.3 must produce an answer that is smaller than either of them.

A second type of argument for decimals being easier than fractions is that differences between whole numbers and fractions make fractions difficult to understand. Researchers making this argument often cite the *whole number bias,* the tendency to assume that the properties of whole numbers apply to all numbers (DeWolf et al. 2014; Ganor-Stern 2013; Ni and Zhou 2005). For example, Ganor-Stern claimed that, "this whole number bias is viewed as an obstacle for learning fractions, and as being responsible at least in part for the poor performance of children in tasks requiring fraction processing" (p. 299). She further argued that, "for them (decimals), there is a positive linear relation between the components magnitude and the holistic magnitude of the number. There is no whole number bias as the whole and the components go in the same direction" (p. 305).

However, although most published illustrations of the whole number bias have been with fractions (e.g., 2/3 + 3/4 = 5/7), the bias applies to decimals as well. Natural numbers (whole numbers other than 0) have unique predecessors and successors, are represented by a single unique symbol within a given symbol system (e.g., "6" or "six"), never decrease with multiplication, and never increase with division. As has often been noted, none of these properties is true for fractions (Gelman 1991; Ni and Zhou 2005). Less often noted, however, none of the properties is true for decimals either. In addition, decimals frequently elicit another manifestation of whole number bias that has not been documented with fractions. In comparing decimals, many children consistently judge longer trains of digits to represent greater magnitudes than shorter trains, for example claiming that 0.123 > 0.45 (Nesher and Peled 1986; Resnick et al. 1989). This reasoning is accurate for whole numbers but not for decimals.

Thus, although whole number and decimal notations are more similar than whole number and fraction notations, many properties of decimals may not be easier to understand than those of fractions. Rather, the difficulty of learning fractions might be general to understanding all types of rational numbers, decimals and percentages as well as fractions. In the following section, we review data on whether learning about decimals is, in fact, easier than learning about fractions. We discuss understanding of percentages in a separate section, because no evidence could be found for percentages regarding most of the topics in the broader discussion.

Difficulties in Understanding Fractions and Decimals

Rational number understanding includes several types of knowledge, among them understanding of magnitudes, arithmetic, density, and translation among formats. In this section, we compare knowledge of each type for fractions and decimals.

Magnitudes. Many people have difficulty understanding fraction magnitudes. With a bipartite structure, fractions can easily be interpreted as two whole numbers, rather than as a single number (Gelman 1991; Mack 1995). In tasks designed to assess knowledge of fraction magnitudes, many people, especially those with relatively low math achievement, tend to only consider the numerator or only the denominator of fractions (Behr et al. 1984; Braithwaite and Siegler, in press; Siegler and Pyke 2013; Stafylidou and Vosniadou 2004).

In one study that examined understanding of fraction magnitudes, Siegler and Pyke (2013) presented 6th and 8th graders a fraction number line estimation task in which the children needed to locate each of a set of fractions on a 0-1 number line. The data analyses included correlating the rank orders of each fraction's numerator, denominator, and overall magnitude with the rank order of each child's estimates of the fractions' sizes. Numerators or denominators, rather than fraction magnitudes, were most highly correlated with many children's estimates, especially children who scored poorly on standardized math achievement tests. Size of the numerator or denominator was a better predictor of estimates than was fraction magnitude for 76% of low-achieving 6th graders and 55% of low-achieving 8th graders (low achievers were defined as children whose standardized achievement test scores were below the 35th percentile).

Similar phenomena have been found on other fraction magnitude tasks. Interviews conducted early in a teaching experiment revealed that when comparing the magnitudes of two unit fractions, most 4th graders viewed fractions as two independent whole numbers (Behr et al. 1984). Similarly, when 5th graders were asked to identify the smallest or biggest of several fractions, 40% indicated that the magnitude of a fraction is determined either by its numerator alone or by its denominator alone (Stafylidou and

Vosniadou 2004). Although holistic processing of fractions is achieved by some older students, the tendency to access the magnitudes of the numerators and denominators in fraction magnitude tasks persists among college students and even math experts (Kallai and Tzelgov 2011; Meert et al. 2009; Obersteiner et al. 2013; Vamvakoussi et al. 2012).

These well-documented difficulties with fraction magnitude understanding are much of the evidence for the view that decimals are easier to understand than fractions. Unfortunately, however, children's understanding of decimals shows similar weaknesses. In decimal comparison, two incorrect rules used by children have been documented in several studies (Desmet et al. 2010; Durkin and Rittle-Johnson 2015; Nesher and Peled 1986; Resnick et al. 1989; Sackur-Grisvard and Léonard 1985). The *whole number rule* posits that with decimals as with whole numbers, the number with the longer train of digits is inevitably larger. This rule leads to errors such as claiming that 0.146 > 0.46. Another fairly common incorrect rule for comparing decimals, the *fraction rule,* appears to reflect superficial understanding of the base-10 system and is probably formed while learning about fractions. Having learned that tenths are bigger than hundredths and hundredths are bigger than thousandths, some students conclude that decimals that express the number of hundredths are smaller than decimals that express the number of tenths, regardless of the numbers involved. These children might reason that 0.47 is smaller than 0.2 because the former reads "forty-seven hundredths," the latter reads "two tenths," and hundredths are smaller than tenths (Nesher and Peled 1986; Resnick et al. 1989).

Errors produced by these two rules are common (Desmet et al. 2010; Durkin and Rittle-Johnson 2015; Nesher and Peled 1986; Resnick et al. 1989; Sackur-Grisvard and Léonard 1985). To cite one example, in Nesher and Peled (1986), 6th graders were asked to state strategies for comparing the magnitudes of pairs of decimals. The problems were designed to detect use of the two rules. A student following the whole number rule would correctly choose 3.47 as larger than 3.2 but incorrectly choose 4.63 as larger than 4.8. In contrast, a student following the fraction rule would correctly choose 4.8 as larger than 4.63 but incorrectly choose 3.2 as larger than 3.47. Using items like these, Nesher and Peled found that the reasoning of 33% of 6th graders about which decimal was bigger consistently followed the fraction rule, and the reasoning of 20% consistently conformed to the whole number rule.

Similar strategies are evident on other tasks that assess decimal magnitude understanding. When 5th graders were asked to choose which of four marks represented a given decimal on a 0 to 1 number line, 39% tended to follow the rule "longer decimals are larger" (Rittle-Johnson et al. 2001). Similarly, Durkin and Rittle-Johnson (2015) presented 4th and 5th graders with tasks assessing number line estimation, magnitude comparison, and density (e.g., write a decimal that comes between 0.14 and 0.148), as well as a task assessing the meaning of zero within decimals (e.g., circle all the numbers that equal 0.51: 0.5100, 0.051, 0.510, 51). Proportions of responses conforming to each rule were calculated by dividing the number of instances consistent with a particular rule by the total number of instances where that rule was applicable. Before any intervention, 42% of the answers of 4th and 5th graders conformed to the whole number rule and 10% conformed to the fraction rule. A similar pattern was found on a separate problem in the same study where students were asked to judge whether 0.□ or 0.□□□ was greater (the

"□"s represent numbers covered by pieces of paper): 52% of answers were consistent with the whole number rule, and 12% were consistent with the fraction rule. Similar to fractions, although frequency of such errors decreases with age and many people correctly compare decimals by accessing their holistic magnitudes, processing of component magnitudes (i.e., magnitudes of the fractional part of decimals) persists even among adults (Ganor-Stern 2013; Varma and Karl 2013).

Studies that have assessed knowledge of the same participants or participants from the same sample provide mixed evidence for the relative ease of understanding decimal and fraction magnitudes. Several studies suggest that people understand decimal magnitudes better than fraction magnitudes. For example, Iuculano and Butterworth (2011) tested 6th graders and college students with decimal and fraction zero to one number line estimation tasks. The assessment of number line estimation included both a position to number task (PN; given a position of a number on a number line, estimate the number) and a number to position task (NP; given a number, estimate its position on a number line). Estimates of fractions were less accurate than those of decimals on both tasks. Moreover, on the PN task, both children's and adults' numerical estimates showed a linear trend with decimals but not fractions. Wang and Siegler (2013) also found that 4th and 5th graders' number line estimation and magnitude comparison accuracy was higher for decimals than fractions.

College students' magnitude comparisons have also been found to be faster and more accurate, and to show a more consistent distance effect with decimals than with fractions (DeWolf et al. 2014; Ganor-Stern 2013; Hurst and Cordes 2016). For example, in Ganor-Stern (2013), a distance effect (quicker and more accurate responses when the numbers compared were further apart) was more consistently present when students compared decimals than when they compared fractions. Similarly, Hurst and Cordes (2016) found that college students were fastest in comparing pairs of decimals, next fastest in comparing one decimal and one fraction, and slowest in comparing two fractions.

In contrast to studies that found a decimal advantage, DeWolf et al. (2015a) found that seventh graders' accuracy on number line estimation was similar for fractions and decimals. Percent absolute error (PAE: (|estimate – correct answer|/numerical range) * 100) was 15% for both types of numbers.

However, a closer look at the decimal and fraction stimuli used in these studies suggests that the presence of a decimal advantage may depend on the equality of the number of digits in the decimals presented to participants. In almost all of the magnitude comparison experiments where a decimal advantage was found, the decimals in each pair had the same number of digits to the right of the decimal point (Experiment 1 and 2 in DeWolf et al. 2014; Ganor-Stern 2013; Hurst and Cordes 2016; Wang and Siegler 2013). Experiment 3 in DeWolf et al. (2014) is the only experiment that found a decimal advantage when the decimals being compared varied in their number of digits. Consistent with this analysis, decimal comparison problems with decimals of unequal numbers of digits pose greater difficulty for children than problems with decimals of equal numbers of digits (Desmet et al. 2010; Durkin and Rittle-Johnson 2015; Nesher and Peled 1986; Rittle-Johnson et al. 2001; Sackur-Grisvard and Léonard 1985).

There is a simple reason why strong performance on comparison tasks involving decimals with equal numbers of digits to the right of the decimal point may not indicate strong understanding of decimal magnitudes. Accurate performance can be produced on such problems by ignoring the decimal point and treating the numbers being compared as whole numbers. Comparisons among the accuracy and speed of magnitude comparison of fractions, decimals with equal numbers of digits to the right of the decimal point, and decimals with unequal numbers of digits to the right of the decimal point, matched for the magnitudes of the numbers being compared, are necessary to establish whether decimal magnitude comparison is generally easier than fraction magnitude comparison.

Similarly, the higher accuracy with decimals than fractions on number line estimation tasks may depend on the number of digits in the decimals. In both of the number line estimation experiments where a decimal advantage was present, all decimals had two digits to the right of the decimal point (Iuculano and Butterworth 2011 (Iuculano, personal communication 13 February 2017); Wang and Siegler 2013). In contrast, the only number line estimation experiment that used decimals of varying number of digits showed identical accuracy for decimals and fractions (DeWolf et al. 2015a). Moreover, recently collected data suggest that children's estimates of decimals to the hundredth place were more accurate than estimates of decimals to the tenth place (Tian and Siegler 2017).

The influence of equality of number of digits on understanding decimal magnitudes decreases with age, as shown by data on decimal magnitude comparison tasks. Sackur-Grisvard and Léonard (1985) found that the percent of students who believed longer decimals are inevitably larger decreased between Grade 4 and 5. Similarly, cross-sectional data from Nesher and Peled (1986) showed that the percent of students holding this belief decreased from Grade 7 to Grade 9. Thus, the age and experience of participants might influence the relative ease of understanding decimal and fraction magnitudes, a hypothesis that could be tested by examining changes with age in relative knowledge of decimal and fraction magnitudes in the same sample.

To summarize, children have different types, but similar levels, of difficulty with fraction and decimal magnitudes. Most studies that found better performance with decimals than fractions have used particularly easy decimal problems: magnitude comparison problems in which both numbers being compared have two digits to the right of the decimal point, and 0-1 number line estimation problems in which the decimals being estimated have two digits to the right of the decimal point (DeWolf et al. 2014; Ganor-Stern 2013; Hurst and Cordes 2016; Iuculano and Butterworth 2011; Wang and Siegler 2013). These decimal comparisons can be answered correctly by treating the decimals as whole numbers, and in the case of number line estimation, by also treating the end points of the number lines as 0 and 100. Thus, good performance on such tasks may not indicate good understanding of decimal magnitudes more generally.

Arithmetic. The literature on rational number arithmetic shows a similar pattern to that on magnitudes. Although misunderstandings of fraction arithmetic have received much more attention, misunderstandings of similar seriousness and prevalence are present in decimal arithmetic. Moreover, a basic reason for the prevalence of these errors – weak conceptual understanding of the arithmetic operations in the context of rational numbers - appears to underlie the errors in both fraction and decimal arithmetic. In this section, we first review findings regarding fraction arithmetic and then ones on decimal arithmetic.

As with misunderstandings of individual fractions, misunderstandings of fraction arithmetic often derive from viewing each operand as two independent whole numbers. Thus, children often produce *independent whole number errors* in fraction addition and subtraction by applying the arithmetic operation independently to the two numerators and to the two denominators. A child using this strategy on 1/2 + 1/3 would answer "2/5".

Independent whole number errors are common in fraction addition and subtraction. For example, in Hecht (1998), independent whole number errors accounted for 39% of the errors that seventh and 8th graders made on fraction addition problems. In Siegler and Pyke (2013), 6th and 8th graders made independent whole number errors on 26% of the addition problems and 20% of the subtraction problems. Community college students and even pre-service teachers also often make such errors (Silver 1983; Stigler et al. 2010).

Beyond the influence of whole number knowledge, difficulty in fraction arithmetic also arises from the complicated relations among the procedures of different fraction arithmetic operations (Lortie-Forgues et al. 2015). Each fraction arithmetic operation comprises a chain of steps; some steps in one arithmetic operation are shared by one or more other fraction arithmetic operations. For example, to solve a fraction addition problem with unequal denominators, one needs to 1) find a common denominator, 2) transform the operands to fractions with that common denominator, 3) add the numerators, and 4) maintain the common denominator in the answer. Step 4 is also a component of fraction subtraction, but not of fraction multiplication or division.

Many children import steps of other fraction arithmetic operations that are inappropriate for the requested operation, a type of mistake that Siegler and Pyke (2013) labeled *wrong fraction operation errors.* For example, maintaining the common denominator in 2/5 *3/5 = 6/5 is a wrong fraction operation error, because it treats the denominator as would be appropriate in an addition or subtraction problem but not in a multiplication problem. In Siegler and Pyke (2013), 6th and 8th graders made wrong fraction operation errors on 55% of the fraction division problems and 46% of the multiplication problems. These errors were also common in other samples of children (Siegler et al. 2011; Torbeyns et al. 2015) and among pre-service teachers (Newton 2008).

Decimal arithmetic performance also is influenced by overgeneralizing whole number arithmetic procedures. For addition and subtraction of decimals with different number of digits in the fractional parts, incorrect alignment of the decimal operands is the most frequent source of errors (Hiebert and Wearne 1985, 1986; Lai and Murray 2014). Children tend to align the rightmost digit of decimals, which is correct in whole number arithmetic and when adding or subtracting decimals with equal numbers of digits to the right of the decimal point, but such alignment is incorrect in decimal addition or subtraction with unequal numbers of digits to the right of the decimal point. For example, when adding 6 and 0.32, 43% of 5th graders answered 0.38 (Hiebert and Wearne 1985). This answer occurred significantly more often than the next most frequent error, a pattern that persisted in 6th, seventh, and ninth grades (Hiebert and Wearne 1985). Similarly, failure to align the decimal points accounted for about half of the errors in decimal addition and subtraction problems among Australian 12-year-olds (Lai and Murray 2014).

Children also have great difficulty correctly placing the decimal point when answering multiplication and division problems (Hiebert and Wearne 1985, 1986; Lai and Murray 2014; Lortie-Forgues and Siegler 2017). For example, when multiplying 0.05 by 0.4, more than half of 6th graders answered 0.2 or 2; these two answers continued to be the most common errors through Grade 9 (Hiebert and Wearne 1985, 1986). Similar findings have been obtained on standardized tests with large, nationally representative samples; on the second NAEP, about half of the decimal multiplication errors made by 13-year-olds involved incorrect placement of the decimal point (Carpenter et al. 1981). In a more recent study, misplacement of the decimal point in the answers accounted for 73% of middle school students' decimal multiplication errors (Lortie-Forgues and Siegler 2017).

The frequencies of errors caused by misalignment of decimal operands in addition and subtraction and by misplacement of the decimal point in multiplication and division answers decrease with age (Hiebert and Wearne 1985, 1986). For example, in Hiebert and Wearne (1986), frequency of misaligning the rightmost digit when solving the problem "4 + .3" decreased from 90% among 4th graders to 84% among 5th and 6th graders, 53% among seventh graders, and 14% among ninth graders.

In both fraction and decimal arithmetic, errors produced by faulty procedures are well documented. Some of these errors violate the direction of effects principle; that is, the magnitude of the answer is in the wrong direction relative to the operands and operation. For example, 2/5 * 3/5 = 6/5 is a direction of effects error, because multiplying two positive numbers smaller than one must yield answers smaller than either operand, and 6/5 is bigger than either operand. However, it is unclear whether such errors are due to lack of conceptual understanding of the arithmetic operations or to heavy working memory demands preventing children from accessing their conceptual understanding while solving rational number arithmetic problems.

The direction of effects task was developed to distinguish between these two theoretical interpretations. On a direction of effects task, participants are asked to evaluate mathematical inequality problems, for example, to judge whether the inequality "31/44 * 27/65 > 31/44" is true or false. Participants are instructed not to compute the arithmetic problems but rather to reason out whether the inequality is true or false. Computation is further discouraged by presenting items that are hard to mentally compute, and not providing paper, pencils, or calculators. Thus, direction of effects problems greatly reduce working memory demands in rational number arithmetic and enable assessment of conceptual understanding of arithmetic.

Siegler and Lortie-Forgues (2015) presented direction of effects problems with fractions to middle school students and pre-service teachers. Accuracy was near ceiling on all addition and subtraction problems and on multiplication and division problems with fractions larger than one. Note that correct answers to these problems are the same as on parallel problems involving whole number operands. For example, "51/16 ÷ 47/33 > 51/16" is "false," just as "51 ÷ 47 > 51" would be.

In contrast, accuracy was below chance on multiplication and division direction of effects problems with fractions between zero and one, where correct answers to the inequalities are inconsistent with correct answers to parallel whole number problems. For example, the inequality "31/56 * 17/42 > 31/56" is "false," unlike parallel multiplication problems involving whole number operands, such as "31 * 17 > 31." Middle school students were correct on only 31% of multiplication and 47% of division items on such problems.

Even more striking, pre-service teachers were correct on only 33% and 30% of these two-choice multiplication and division direction of effects problems with fraction operands between zero and one. In contrast, the pre-service teachers, like the middle school students, were correct on more than 90% of judgments on the six types of problems where the correct answers were the same as it would have been if the operands were whole numbers. The multiplication and division problems with fractions between zero and one were far from impossible; students at a highly selective university were correct on 94% to 100% of all eight types of items.

Performance on direction of effects tasks with decimals revealed the same type of weak understanding as found with fractions. In Lortie-Forgues and Siegler (2017), 6^{th} and 8^{th} graders were correct on only 19% of both multiplication and division direction of effects problems with decimal operands between zero and one. In contrast, the same children were correct on almost 90% of direction of effects multiplication and division problems with decimal operands above one. Thus, many people seem to hold the misconception that "multiplication makes bigger" and "division makes smaller, " a generalization that they seem to apply to both fractions and decimals regardless of operand magnitudes (Fischbein et al. 1985; Graeber and Tirosh 1990).

To summarize, fraction and decimal arithmetic pose similarly large difficulties to learners. Errors with both notations are common among children and even adults, and it is unclear whether arithmetic with either notation is generally easier than with the other. Moreover, conceptual understanding of rational number arithmetic is similarly weak with both notations, as evidenced by highly similar patterns of correct answers and errors on fraction and decimal direction of effects problems. This weak conceptual understanding probably underlies the many particular arithmetic errors seen with both fractions and decimals.

Density. Density understanding involves knowing that there are an infinite number of fractions and decimals between any two other numbers. Both high school students and younger children seem to have little understanding of the density of either fractions or decimals. When answering questions regarding whether there are a finite or an infinite number of numbers between two decimals or fractions, the answer "finite" was the most common response among ninth graders in both open-ended (65%) and forced-choice (52%) questions. The percent of children showing this misconception decreased during high school, but more than one-third of eleventh graders answered "finite" to both questions (Vamvakoussi and Vosniadou 2007).

This study did not report performance on fraction and decimal problems separately, but Vamvakoussi and Vosniadou (2010) did. They tested seventh, ninth, and eleventh graders' knowledge of density with pairs of decimals with the same number of digits to the right of the decimal point and pairs of fractions with the same denominator or the same numerator on force-choice questions. For each question, students' performance was scored "1" if they chose the options that there are a finite number of decimals or fractions between the two numbers, "2" if they chose the options that there are an infinite number of one type of number between the two numbers (e.g., "there are infinitely many decimals"), or "3" if they choose the option that there are an infinite number of all types of numbers between them (i.e., "there are infinitely many numbers and they have various forms").

Mean performance was around 1.5 for seventh graders and around 1.9 for ninth and eleventh graders. Thus, even many eleventh graders had not reached a correct understanding of the density property of decimals or fractions. When comparing performance on density problems for the two notations, seventh graders performed better on decimals than fractions, but the judgments of ninth and eleventh graders did not differ on the two types of problems.

In Vamvakoussi et al. (2011), Greek and Flemish ninth graders were tested with the same problems as in Vamvakoussi and Vosniadou (2010). As in the prior study, the density judgments of students in Vamvakoussi et al. (2011) were similar on decimal and fraction pairs.

Many adults also have little understanding of the density property of decimals and fractions. Tirosh et al. (1999) found that only 40% of Israeli pre-service teachers knew that there are an infinite number of numbers between 0.23 and 0.24, and only 24% knew that there are an infinite number of numbers between 1/5 and 1/4. Unlike the better performance on decimal than on fraction density judgments found in Tirosh et al. (1999), Giannakoulias et al. (2007) found that more college students knew that there are an infinite number of numbers between two fractions (87%) than between two decimals (65%). Thus, as with knowledge of magnitudes and arithmetic, there does not seem to be a consistent difference between understanding of density for decimals and fractions.

Translation. Given low levels of accuracy on tasks involving only fractions or only decimals, it is not surprising that many children have difficulty viewing decimals and fractions as alternative notations within a single unified system (O'Connor 2001; Pagni 2004; Sweeney and Quinn 2000; Vamvakoussi and Vosniadou 2010). For example, when asked about the type of numbers between two fractions or two decimals, nearly 80% of seventh graders and more than half of ninth and eleventh graders indicated that that there are only decimals between two decimals and only fractions between two fractions (Vamvakoussi and Vosniadou 2010).

Children's lack of understanding that fractions and decimals are alternative notations for expressing magnitudes and other numerical properties is also revealed by their poor performance translating between the two notations. Hiebert and Wearne (1983) examined 5th, seventh, and ninth graders' translations of decimals to fractions and fractions to decimals. Accuracy of 5th graders was between 19% and 31% correct when translating fractions between zero and one with a denominator of 10 or 100 into decimals, far from good but better than the roughly 10% correct when translating decimals between zero and one with two digits to the right of the decimal point into fractions (only exact answers were considered correct; Hiebert, personal communication 22 February 2017). Nearly one fourth of the 5th graders simply transformed the numerals in fractions or decimals to another format (e.g. 0.37 = 3/7; 4/10 = 4.10).

Accuracy of seventh and ninth graders was considerably greater than that of 5th graders, but understanding remained limited (Hiebert and Wearne 1983). Accuracy of translating proper fractions with a denominator of 10 or 100 into decimals was about 70%

among seventh graders and 85% among ninth graders. However, interviews with a representative sample of the seventh and ninth graders indicated that more than half of the seventh graders and more than one third of the ninth graders "were unable to write ¼ as a decimal" (Hiebert and Wearne 1985, p. 23). Most seventh and ninth graders knew that a decimal could be translated into a fraction with a denominator of a power of ten, and that such a fraction could be translated into a decimal by using the numerator and placing the decimal point at a particular position (the exact percentage of children with this knowledge was not included in the report). However, fewer students could explain why these procedures worked. Similarly, although about half of 8th graders accurately translated fractions with a denominator of 10 or 100 into decimal equivalents on a NAEP, less than 40% correctly chose " .2" as equivalent to 1/5 from among 0.15, 0.2, 0.5, 0.51 and "I don't know" (Carpenter et al. 1981).

An interview with a 12-year-old boy named Benny provided a detailed illustration of the tendency of many children to apply a rule for converting fractions to decimals without any obvious conceptual basis (Erlwanger 1973). Benny's procedure for converting a fraction into a decimal was to find the sum of the numerator and the denominator of the fraction and then place the decimal point to the left of the sum if the sum was a single digit (e.g., 1/8 = .9) or to place the decimal point immediately to the right of the leftmost digit of the sum if the sum included multiple digits (e.g., 9/10 = 1.9). Benny was confident in his answers and followed his rules consistently.

Many adults also show limited understanding of equivalent fractions and decimals. When asked to choose all numbers equivalent to 0.03 from a list of rational numbers, more than half of community college students correctly chose 3/100 and 3%, but only 9% chose 30/1000 (Stigler et al. 2010). Thus, accuracy and strategy reports on translation problems suggest that translating between fractions and decimals is difficult for many children and adults. One potential explanation is that they might not realize that decimals and fractions are part of a single system of rational numbers.

Understanding of Percentages

Much less is known about development of understanding of percentages than about development of understanding of either fractions or decimals. The relatively few studies are often not directly comparable to those that have been done with fractions and decimals, and much of the little that is known comes from studies conducted more than 70 years ago, in the 1940's. We present what is known in this section, so that the review includes all three rational number notations and to provide a foundation for more extensive future research on this important topic.

In the U.S., percentages usually are formally introduced in Grade 6, which is later than decimals and fractions (CCSSI, 2010). Children do have informal experience with percentages from earlier ages, though. For example, percentages are pervasively used to indicate the materials in clothing (e.g., a 100% silk dress) and to represent price discounts (e.g., a 40% off sale).

Similar to decimals and fractions, children's understanding of percentages is limited. Some children showed competence with familiar percentages, such as 50%, 100%, or 25%, but not with unfamiliar ones. Gay and Aichele (1997) asked seventh and 8th graders to match a shaded part of a rectangle or a set of circles to a given percentage. The percentages used were 50%, 25%, 100%, 60%, 110%, 33 1/3%, and 87%, presented in order from the most familiar to the least familiar, as determined by a pilot study. For example, one question included three shaded circles and two blank ones. Children were asked to decide whether, compared to the whole set, the shaded circles were "greater than 50%", "less than 50%", "equal to 50%", "can't tell", or "I don't know". Children judged more accurately when the percentage was familiar than when it was unfamiliar. Similarly, Lembke and Reys (1994) indicated that fifth, seventh, ninth and eleventh graders used familiar percentages (e.g., 50%, 25%, and 100%) as benchmarks to identify unfamiliar percentages of squares, lines, and circles.

Children's arithmetic involving percentages is quite inaccurate. Much of the little that is known about this topic has been learned on "Percentage * Whole = Portion" problems. Participants were asked to either find percentage, whole or portion when the other two were known. In one study (Guiler 1946a), only about half of ninth graders found the percentage when the whole and the portion were known (e.g., 20 games is __% of 25 games), or found the portion when the percentage and the whole were known (e.g., 88% of $1.75 = $ __). When asked to find the whole given the percentage and the portion (e.g., 125% of $__ = $8.00), only 6% of ninth graders responded correctly. College freshmen were similarly inaccurate on the same problems. For example, only 12.3% found the whole given the percentage and the portion (Guiler 1946b). These studies were conducted more than 70 years ago, and arithmetic with percentages might have improved in the interim, but given the general secular decline in numerical skills among American adults (Geary et al. 1996, 1997; Schaie 1993), this seems far from certain.

Analyses of ninth graders' and college freshmen's errors on the problems revealed that using inappropriate procedures was a common mistake when finding the percentage or the whole (Guiler 1946a, 1946b). To cite an example, 18% of ninth graders and 15% of college freshmen divided the whole by the portion to find the percentage (e.g., to find "20 games is _% of 25 games", they calculated 25/20 = 125%). To cite another example, 36% of ninth graders and 32% of college freshmen multiplied the percentage and the portion to find the whole.

Similar to decimals and fractions, many children do not understand the direction of effects of multiplication with percentages between zero and one (Gay and Aichele 1997). Only 45% of seventh and 8th graders responded correctly when asked whether 87% of 10 was less than 10, greater than 10, equal to 10, "can't tell", or "I don't know" (Gay and Aichele 1997). Among the 55% of children who were unsuccessful on this problem, 61% chose "greater than 10", which is consistent with the prevalent belief among many children that "multiplication makes bigger" (Fischbein et al. 1985; Graeber and Tirosh 1990). Similarly, on the 1986 NAEP, when asked whether 76% of 20 is greater than, less than, or equal to 20, only 37% of seventh graders and 69% of eleventh graders answered correctly (Kouba et al. 1988). Thus, the few studies that have been conducted on arithmetic with percentages indicate that even high school and college students' knowledge in this area is weak.

Rational Number Interventions

A more direct way to answer the question "which rational number format should be taught first" is to assess the effects of interventions that implement different teaching sequences._One intervention that addressed this issue contrasted the effects of a widely used Canadian curriculum that taught rational numbers in the standard order of fractions-decimals- percentages with the effects of an experimental curriculum that taught the three in the opposite order: percentages-decimals-fractions (Moss 1997; Moss and Case 1999). The participants, fourth graders without any previous instruction in rational numbers, were randomly assigned to receive one of the two curricula. The experimental curriculum included 20 sessions over a 5-month period; the traditional one included 25 sessions over a slightly shorter time period.

The experimental curriculum emphasized connections among the three rational number formats. Instruction started with an introduction to percentages, with students being asked to estimate the fullness of several glasses of water in percentages. Two-digit decimals were later introduced as an alternative notation to percentages. Fraction terminology was used informally throughout the program as an alternative notation to percentages and decimals before being formally introduced. For example, the term one half or ½ was used interchangeably when 50% or 0.50 was presented. At the end of the intervention, lessons focused on fractions were given. In them, children represented fractions in multiple ways, including spatial displays, decimals, and percentages, and solved equations involving fractions as well as decimals and percentages.

After the intervention, children in the experimental group demonstrated greater understanding of rational numbers on a variety of measures, including: non-standard computation, standard computation, symbol-graphic representation, magnitude knowledge, word problems, and translation among notations. On the assessment, the authors "intentionally included a number of questions that were closer in their content to the sort of training that the experimental group received and a number that were closer to the training received by the control group" (Moss and Case 1999, p. 134). Children in the experimental group were similarly accurate on standard computation to children in the control group, although they, unlike children in the control group, did not receive instruction on standard rational number arithmetic algorithms. Compared with children in the control group, children in the experimental group also relied less on whole number strategies when solving novel problems and were more accurate in translating fractions into decimals. Later studies implemented this rational number curriculum with another group of fourth graders and a group of sixth graders; both groups achieved better learning results compared to students who received traditional curricula (Kalchman et al. 2001).

Moss and Case (1999) argued that the teaching sequence of the three rational number notations that was implemented in the intervention "maximize the connection between their (i.e., children's) original, intuitive understanding of ratios and their procedures of splitting numbers" (p. 126). Because this experimental curriculum differed in many ways from the control curriculum, not only in the order in which rational number formats were introduced but also in its greater emphasis on magnitudes and its emphasis on linking the curricular materials to children's intuitive understanding, the effect of the sequence in which rational numbers were taught remains to be established. However, the positive effects of this intervention justify further examination of whether changing the order in which rational number formats are taught would result in improved learning.

Discussion and Conclusions

Are decimals easier to understand than fractions? The shared base-10 structure of whole number and decimal notations, together with the quite different bipartite structure of fractions, have led a number of researchers to propose that decimals are easier to master than fractions (DeWolf et al. 2014, 2015b; Ganor-Stern 2013; Hurst and Cordes 2016; Iuculano and Butterworth 2011; Johnson 1956; Zhang et al. 2013). However, the present review indicates that this conclusion is not justified. Although the symbols used to represent decimals and whole numbers have clear similarities, the same conceptual difficulties that interfere with understanding of fractions also interfere with understanding of decimals, leading to similarly weak understanding of fractions and decimals.

The empirical basis for arguing that decimals are easier to understand than fractions comes largely from studies of magnitude comparison. In these studies, performance on problems involving decimals has been found to be more accurate and faster than performance on problems involving fractions (DeWolf et al. 2014; Hurst and Cordes 2016; Wang and Siegler 2013). However, these studies have confounded notation with denominator equality. The fraction problems have had unequal denominators, whereas the decimal problems have had the equal denominators that are implicit when decimals have the same number of digits to the right of the decimal point. Comparing decimals with equal numbers of digits to the right of the decimal point allows participants to perform quickly and accurately by simply ignoring the decimal point and treating the decimals as whole numbers.

Thus, in magnitude comparison tasks, performance on relatively easy decimal problems (ones with equal numbers of digits to the right of the decimal point) has been compared to performance on relatively difficult fraction problems (ones with unequal denominators). Attesting to the decimal comparison problems being relatively easy, magnitude comparison of decimals is more accurate when the decimals being compared have equal rather than unequal numbers of digits to the right of the decimal point (Desmet et al. 2010). Attesting to the fraction comparisons problems being relatively difficult, magnitude comparisons of fractions are slower and less accurate when the fractions have unequal rather than equal denominators (Obersteiner et al. 2013).

This confounding of notation with problem difficulty on magnitude comparison problems implies that appropriate conclusions regarding the difficulty of understanding fraction and decimal magnitudes will require two types of comparisons that have not yet been performed. One informative comparison would be between easy problems in each notation (magnitude judgments involving pairs of decimals with equal numbers of digits to the right of the decimal point being compared to magnitude judgments with pairs of fractions with equal denominators). Another informative comparison would be between difficult types of problems in each notation (magnitude judgments involving pairs of decimals with unequal numbers of digits to the right of the decimal point being compared to magnitude judgments involving pairs of fractions with unequal denominators). Including all four types of comparison in a single study would allow examination of interactions between notation and problem difficulty, as well as of whether one notation was generally easier.

Similarly, the number line estimation tasks where children performed better with decimals than fractions used easy decimal problems (ones in which all decimals had two digits to the right of the decimal point) and difficult fraction problems (ones with varying denominators) (Iuculano and Butterworth 2011; Wang and Siegler 2013). The one number line study that compared fraction and decimal magnitude knowledge using difficult problems for both notations (DeWolf et al. 2015a) did not show a difference between the accuracy of estimates for the two notations; percent absolute error was identical for the two notations (both 15%).

With regard to other areas of research on knowledge of fractions and decimals -- arithmetic procedures, conceptual understanding of arithmetic, and the density property of rational numbers -- lack of research assessing fraction and decimal knowledge of the same participants or participants from the same sample precludes direct comparisons of understanding of the two notations. However, similar misconceptions are evident with both notations, and their frequency seems roughly comparable. Children's poor performance and frequent errors on both fraction and decimal arithmetic are well documented (Hiebert and Wearne 1985, 1986; Lai and Murray 2014; Lortie-Forgues et al. 2015; Siegler and Pyke 2013). With both fractions and decimals, weak conceptual understanding of arithmetic with rational numbers seems to underlie diverse particular errors. Many, probably most, children and adults mistakenly believe that multiplying two fractions or decimals always produces answers greater than the operands and that dividing by a fraction or a decimal always yields an answer smaller than the dividend (Lortie-Forgues and Siegler 2017; Siegler and Lortie-Forgues 2015).

Children's understanding of the density property of both fractions and decimals is also weak. They often deny that there are an infinite number of numbers between fractions such as 5/7 and 6/7 and between decimals such as 0.31 and 0.32 (Vamvakoussi et al. 2011; Vamvakoussi and Vosniadou 2007, 2010). Again, no difference in understanding of this property of fractions and decimals is evident.

Little research has been performed that directly compares development of decimal and fraction knowledge. This is true in all of the areas of development considered in this review: rational number magnitudes, arithmetic, density, and translation. Children's performance improves with age and experience in all areas (Braithwaite and Siegler, in press; Hiebert and Wearne 1983, 1985; Nesher and Peled 1986; Siegler and Pyke 2013; Vamvakoussi and Vosniadou 2010), but the developmental trajectories and asymptotic levels of understanding are impossible to compare without studies presenting comparable fraction and decimal problems (and ideally problems with percentages as well) to participants of a wide age range (ideally the same participants). Such developmental studies seem to merit high priority, because they would substantially improve understanding of the development of rational number knowledge.

Lack of research on knowledge of percentages. Given the importance of percentages and students' poor knowledge of them (Carpenter et al. 1975; Kloosterman 2012; Kouba et al. 1988), it is surprising how little research has been done to investigate them. In this review, we were only able to locate four published articles that focused on knowledge of percentages (Gay and Aichele 1997; Guiler 1946a, 1946b; Lembke and Reys 1994). Moreover, two of the four were published 70 years ago, which renders uncertain their applicability to current student populations. Parker and Leinhardt (1995) conducted a review of "70 years of empirical research" (p. 423) on learning of percentages. However, the data cited in that review were mostly from dissertations, unpublished manuscripts, and conference proceedings, making it difficult to evaluate the quality of the studies.

The poor performance documented in the two published studies from the 1940's is unlikely to have improved, given the general cross-generational decline in numerical skills among American adults (Geary et al. 1996, 1997; Schaie 1993). Thus, remedying poor understanding of percentages remains a serious concern at present. The lack of published articles on the topic, together with the importance of the topic, makes research on children's understanding of percentages essential for a comprehensive depiction of rational number knowledge and its development.

Which rational number format should be learned first? Although existing findings indicate that children have weak understanding of decimals, fractions, and percentages alike, teaching the three rational number formats in a sequence different than that in the traditional curriculum may still be useful. In particular, children's superior learning from the curriculum developed by Moss and her colleague (Kalchman et al. 2001; Moss 1997; Moss and Case 1999) is consistent with the hypothesis that teaching rational numbers in the sequence of percentages first, decimals next, and fractions last yields better outcomes than the traditional sequence.

To decide which rational number format should be taught (and learned) first, several questions need to be considered: 1) which rational number format is easiest to learn, 2) in which rational number format can misunderstandings be overcome most quickly, and 3) from which rational number format can knowledge be transferred most easily. Types of research needed to answer the first two questions have been discussed above. To investigate the third question, research on translation among rational number formats is needed.

The limited work on translating between rational number notations leaves unclear how well knowledge of the first-taught notation is transferred to later-taught ones. Hiebert and Wearne (1983) found that children translated fractions with a denominator of 10 or 100 into decimals more accurately than they translated decimals with two digits to the right of the decimal point into fractions. However, performance on the second NAEP showed weak translation performance between fractions and decimals in both directions (Carpenter et al. 1981). We were unable to locate any research that examined translation between percentages and the other rational number notations in either direction.

Clearly, far more research is needed before data will provide a rational basis for choosing one instructional order over others. To deepen our understanding of numerical development, and to develop more effective instruction on all three rational number notations, we believe at least four issues need to be addressed. First, research on knowledge of percentages, especially on their magnitudes, density, and arithmetic, needs to be conducted. Second, parallel fraction, decimal, and percentage items need to be presented to the same participants to determine the relative ease of understanding of the three notations. Third, such research should be done with young children who are at the beginning of rational number instruction; with older children, who are receiving or have recently received instruction in these topics; and with adolescents and adults, who often use rational number arithmetic in mathematics and science courses and in their work. In this way, the developmental trajectory of knowledge of rational numbers can be better understood. Finally, research on translation among different rational number formats is needed, to examine the degree to which people possess an integrated representation of rational numbers and to identify ways to promote integrated knowledge of them. Together, these four types of research may provide a theoretical and empirical basis for predicting the optimal order for teaching students the three rational number notations, for testing the predictions, and for creating improved instructional methods for helping students understand them.

Instruction in rational numbers. The sequence in which different rational number notations are taught is obviously not the only factor that influences the success of rational number instruction. As discussed above, rational numbers are a complicated construct not only because that they can be expressed in different notations but also because each notation has several interpretations. Even understanding all the interpretations of one rational number notation is not easy for children, much less extending the interpretations from one notation to the others.

There have been many efforts to improve rational number instruction through focusing on a particular interpretation. Steffe (2001) proposed a unit coordination approach that encouraged children to establish a partition scheme to obtain the concept of unit fractions and then to reorganize their whole number counting schemes to construct any given fractions besides unit fractions. Confrey and her colleagues (Confrey and Smith 1994; Lachance and Confrey 1995) suggested that decimal concepts should be based on splitting actions such as sharing, doubling, and halving. In contrast to these constructivist approaches, Kellman et al. (2008) employed a perceptual learning method to teach fractions through improving students' ability to extract structures and patterns from problems involving fractions.

Although all these approaches and many others have improved certain aspects of children's knowledge of rational numbers, little attention has been paid to integrating understanding of the three notations. The goal of rational number instruction is to achieve a robust and flexible understanding of all rational number notations. Thus, efforts should be made not only to improve instructional practices of a particular notation, but also to facilitate transfer of knowledge from one notation to another. We are a long way from this goal, but it is an important goal to pursue.

Acknowledgements:
The research reported here was supported in part by the Institute of Education Sciences, U.S. Department of Education, through Grants R305A150262 and R324C100004:84.324C, Subaward 23149 to Carnegie Mellon University, in addition to the Teresa Heinz Chair at Carnegie Mellon University and the Siegler Center for Innovative Learning and Advanced Technology Center, Beijing Normal University. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.

Compliance with Ethical Standards

Conflict of Interest: The authors declare that they have no conflict of interest.

References:

Australian Curriculum and Assessment Reporting Authority. (2014). *Foundation to Year 10 Curriculum: Mathematics.* http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1. Accessed 25 February 2017.

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). Rational number, ratio, and proportion. In *Handbook of Research on Mathemtics Teaching and Learning* (pp. 296–333). New York: Macmillan.

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational Number Concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes* (pp. 91–125). New York: Academic Press. http://www.discoveryeducation.com/teachers/free-lesson-plans/rational-number-concepts.cfm

Behr, M. J., Wachsmuth, I., Post, T. R., & Lesh, R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for Research in Mathematics Education*, *15*(5), 323–341. doi:10.2307/748423

Blair, K., Rosenberg-Lee, M., Tsang, J. M., Schwartz, D. L., & Menon, V. (2012). Beyond natural numbers: negative number representation in parietal cortex. *Frontiers in human neuroscience*, *6*, 1–17. doi:10.3389/fnhum.2012.00007

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemorary Educational Psychology*, *37*, 247–253.

Booth, J. L., Newton, K. J., & Twiss-Garrity, L. K. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, *118*(1), 110–118. doi:10.1016/j.jecp.2013.09.001

Braithwaite, D. W., & Siegler, R. S. (in press). Developmental changes in the whole number bias. *Developmental Science*. https://doi.org/10.1111/desc.12541

Carpenter, T. P., Coburn, T. G., Reys, R. E., & Wilson, J. W. (1975). Results and implications of the NAEP mathematics assessment: Secondary school. *The Mathematics Teacher*, *68*(6), 453–470.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Lindquist, M. M., & Reys, R. E. (1981). Decimals: Results and implications from national assessment. *The Arithmetic Teacher*, *28*(8), 34–37.

Carpenter, T. P., Kepner, H., Corbitt, M. K., Lindquist, M. M., & Reys, R. E. (1980). Results and implications of the second NAEP mathematics assessment: Elementary school. *The Arithmetic Teacher*, *27*(8), 10–47.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a Theoretical Model to Study Students' Understandings of Fractions. *Educational Studies in Mathematics*, *64*(3), 293–316. doi:10.1007/s

Common Core State Standards Initiative (2010). *Common core state standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf. Accessed 25 February 2017.

Confrey, J., & Smith, E. (1994). Exponential Functions, Rates of Change, and the Multiplicative Unit. *Educational Studies in Mathematics*, *26*(2), 135–164.

Cramer, K. A., Post, T. R., & DelMas, R. C. (2002). Initial fraction learning by fourth- and fifth-grade students: A comparison of the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education*, *33*(2), 111–144. doi:10.2307/749646

Department for Education (2013). *The National Curriculum in England: Key Stages 1 and 2 framework document.* https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/425601/PRIMARY_national_curriculum.pdf. Accessed 25 February 2017.

Desmet, L., Grégoire, J., & Mussolin, C. (2010). Developmental changes in the comparison of decimal fractions. *Learning and Instruction*, *20*(6), 521–532. doi:10.1016/j.learninstruc.2009.07.004

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015a). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology*, *133*, 72–84. doi:10.1016/j.jecp.2015.01.013

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015b). Conceptual Structure and the Procedural Affordances of Rational Numbers : Relational Reasoning With Fractions and Decimals. *Journal of experimental psychology. General*, *144*(1), 127–150. doi:10.1037/xge0000034

DeWolf, M., Grounds, M. a., Bassok, M., & Holyoak, K. J. (2014). Magnitude comparison with different types of rational numbers. *Journal of Experimental Psychology: Human Perception and Performance*, *40*(1), 71–82. doi:10.1037/a0032916

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., et al. (2007). School readiness and later achievement. *Developmental Psychology*, *43*(6), 1428–1446. doi:10.1037/0012-1649.43.6.1428

Durkin, K., & Rittle-Johnson, B. (2015). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction*, *37*, 21–29. doi:10.1016/j.learninstruc.2014.08.003

Erlwanger, S. . (1973). Benny's Conception of Rules and Answers in IPI Mathematics. *Journal of Children's Mathematical Behavior*, *1*, 7–26.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, *16*(1), 3–17. doi:10.2307/748969

Fosnot, C. T., & Dolk, M. (2002). *Young Mathematicians at Work, 3: Constructing Fractions, Decimals, and Percents*. Portsmouth, NH: Heinemann.

Ganor-Stern, D. (2013). Are 1/2 and 0.5 represented in the same way? *Acta Psychologica*, *142*(3), 299–307. doi:10.1016/j.actpsy.2013.01.003

Gay, A. S., & Aichele, D. B. (1997). Middle school students' understanding of number sense related to percent. *School Science and Mathematics*, *97*(1), 27–36. doi:10.1111/j.1949-8594.1997.tb17337.x

Geary, D. C., Hamson, C. O., Chen, G.-P., Liu, F., Mary K. Hoard, Salthouse, T. A., et al. (1997). Computational and reasoning abilities in arithmetic: Cross-generational change in China and the United States. *Psychonomic Bulletin & Review*, *4*(3), 425–430. doi:10.3758/BF03210805

Geary, D. C., Hoard, M. K., & Bailey, D. H. (2012). Fact retrieval deficits in low achieving children and children with mathematical learning disability. *Journal of learning disabilities*, *45*(4), 291–307. doi:10.1177/0022219410392046

Geary, D. C., Salthouse, T. A., Chen, G. P., & Fan, L. (1996). Are East Asian versus American differences in arithmetical ability a recent phenomenon? *Developmental Psychology*, *32*(2), 254–262. doi:10.1037//0012-1649.32.2.254

Gelman, R. (1991). Epigenetic foundations of knowledge structures: Initial and transcendent constructions. In S. Carey & R. Gelman (Eds.), *The epigenesis of mind: Essays on biology and cognition* (pp. 293–322). Hillsdale, NJ: Erlbaum.

Giannakoulias, E., Souyoul, A., & Zachariades, T. (2007). Students' thinking about fundamental real numbers properties. In D. Pitta-Pantazi, & G. Philippou (Eds.), *Proceedings of the fifth congress of the European society for research in mathematics education.* Cyrus: ERME, Department of Education, University of Cyprus (pp. 416–425).

Graeber, A. O., & Tirosh, D. (1990). Insights fourth and fifth graders bring to multiplication and division with decimals. *Educational Studies in Mathematics*, *21*(6), 565–588. doi:10.1007/BF00315945

Guiler, W. S. (1946a). Difficulties in percentage encountered by ninth-grade pupils. *The Elementary School Journal*, *46*(10), 563–573. http://www.jstor.org/stable/998710

Guiler, W. S. (1946b). Difficulties encoutered in percentage by college freshmen. *Journal of Educational Research*, *40*(2), 81–95. http://www.jstor.org/stable/27528757

Handel, M. (2016). What do people do at work? A profile of U.S. jobs from the survey of workplace Skills, Technology, and Management Practices (STAMP). *Journal for Labour*

Market Research, *49*(2), 177–197. doi:10.1007/s12651-016-0213-1

Hanson, S. A., & Hogan, T. P. (2000). Computational estimation skill of college students. *Journal for Research in Mathematics Education*, *31*(4), 483–499. doi:10.2307/749654

Hecht, S. A. (1998). Toward an information-processing account of individual differences in fraction skills. *Journal of Educational Psychology*, *90*(3), 545–559. doi:10.1037/0022-0663.90.3.545

Hiebert, J., & Wearne, D. (1983, April). *Students' Conceptions of Decimal Numbers*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Quebec, Canada.

Hiebert, J., & Wearne, D. (1985). A model of students' decimal computation procedures. *Cognition and Instruction*, *2*(3), 175–205. doi:10.1080/07370008.1985.9648916

Hiebert, J., & Wearne, D. (1986). Procedures over concepts: The acquisition of decimal number knowledge. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 199–223). Hillsdale, NJ: Erlbaum.

Hurst, M., & Cordes, S. (2016). Rational-number comparison across notation: Fractions, decimals, and whole numbers. *Journal of Experimental Psychology: Human Perception and Performance*, *42*(2), 281–293. doi:10.1037/xhp0000140

Iuculano, T., & Butterworth, B. (2011). Understanding the real value of fractions and decimals. *Quarterly journal of experimental psychology*, *64*(11), 2088–2098. doi:10.1080/17470218.2011.604785

Johnson, J. T. (1956). Decimal versus common fractions. *The Arithmetic Teacher*, *3*(5), 201–206.

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology*, *116*(1), 45–58. doi:10.1016/j.jecp.2013.02.001

Kalchman, M., Moss, J., & Case, R. (2001). Psychological models for the development of mathematical understanding: Rational numbers and functions. In S. Carver & D. Klahr (Eds.), *Cognition and instruction: Twenty-five years of progress* (pp. 1–38). Mahwah, NJ: Erlbaum.

Kallai, A. Y., & Tzelgov, J. (2011). When meaningful components interrupt the processing of the whole: The case of fractions. *ACTPSY*, *139*(2), 358–369. doi:10.1016/j.actpsy.2011.11.009

Kellman, P. J., Massey, C., Roth, Z., Burke, T., Zucker, J., Saw, A., et al. (2008). Perceptual learning and the technology of expertise Studies in fraction learning and algebra. *Pragmatics & Cognition*, *16*(2), 356–405. doi:10.1075/p

Kieren, T. E. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. In R. Lesh (Ed.), *Number and measurement: Papers from a research workshop* (pp. 101–144). Columbus, OH: ERIC/SMEAC.

Kieren, T. E. (1980). The rational number construct: Its elements and mechanisms. In *Recent Research on Number Learning* (pp. 125–150).

Kloosterman, P. (2010). Mathematics Skills of 17-Year-Olds in the United States: 1978 to 2004. *Journal for Research in Mathematics Education*, *41*(1), 20–51. doi:10.2307/40539363

Kloosterman, P. (2012). Mathematics performance of 13-year olds in the United States: 1978 to 2004., *1*, 1–30. doi:10.1017/CBO9781107415324.004

Koedel, C., & Tyhurst, E. (2012). Math skills and labor-market outcomes: Evidence from a resume-based field experiment. *Economics of Education Review*, *31*(1), 131–140. doi:10.1016/j.econedurev.2011.09.006

Kouba, V. L., Brown, C. A., Carpenter, T. P., Lindquist, M. M., Silver, E. A., & Swafford, J. O. (1988). Results of the fourth NAEP assessment of mathematics: Number, operations, and word problems. *The Arithmetic Teacher*, *35*(8), 14–19.

Lachance, A., & Confrey, J. (1995). Introducing fifth graders to decimal notation through ratio and proportion. *Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.

Lai, M. Y., & Murray, S. (2014). What do error patterns tell us about Hong Kong Chinese and

Australian students' understanding of decimal numbers? *International Journal for Mathematics Teaching & Learning*. http://www.cimt.org.uk/journal/lai2.pdf. Accessed 4 January 2017.

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. New York: Routledge.

Lembke, L. O., & Reys, B. J. (1994). The development of, and interaction between, intuitive and school-taught ideas about percent. *Journal for Research in Mathematics Education*, *25*(3), 237–259. doi:10.2307/749290

Lortie-Forgues, H., & Siegler, R. S. (2017). Conceptual knowledge of decimal arithmetic. *Journal of Educational Psychology*, *109*(3), 374–386. doi:10.1037/edu0000148

Lortie-Forgues, H., Tian, J., & Siegler, R. S. (2015). Why is learning fraction and decimal arithmetic so difficult? *Developmental Review*, *38*, 201–221. doi:10.1016/j.dr.2015.07.008

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates. doi:10.5860/CHOICE.37-2294

Mack, N. K. (1995). Confounding whole-number and fraction concepts when building an informal knowledge. *Journal for Research in Mathematics Education*, *26*(5), 422–441. doi:10.2307/749431

Martin, W. G., Strutchens, M. E., & Elliott, P. C. (Eds.) (2007). *The learning of mathematic,* 69th NCTM yearbook. Reston, VA: National Council of Teachers of Mathematics.

McCloskey, M. (2007). Quantitative literacy and developmental dyscalculias. In D. B. Berch & M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 415–429). Baltimore, MD: Paul H. Brookes Publishing.

Meert, G., Grégoire, J., & Noël, M. (2009). Rational numbers: Componential versus holistic representation of fractions in a magnitude comparison task. *The Quarterly Journal of Experimental Psychology*, *62*(8), 1598–1616. doi:10.1080/17470210802511162

Moss, J. (1997). *Developing children's rational number sense: A new approach and an experimental program*. University of Toronto.

Moss, J., & Case, R. (1999). Developing children's understanding of the rational numbers: A new model and an experimental curriculum. *Journal for Research in Mathematics Education*, *30*(2), 122. doi:10.2307/749607

Nesher, P., & Peled, I. (1986). Shifts in reasoning. *Educational Studies in Mathematics*, *17*(1), 67–79. doi:10.1007/BF00302379

Newton, K. J. (2008). An extensive analysis of preservice elementary teachers' knowledge of fractions. *American Educational Research Association*, *45*(4), 1080–1110. doi:DOL 10.3102/0002831208320851

Ni, Y., & Zhou, Y.-D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*(1), 27–52. doi:10.1207/s15326985ep4001_3

O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?" A case study of a mathematical group discussion. *Educational Studies in Mathematics*, *46*(1), 143–185.

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64–72. doi:10.1016/j.learninstruc.2013.05.003

Pagni, D. (2004). Fractions and decimals. *Australian Mathematics Teacher*, *60*(4), 28–30.

Parker, M., & Leinhardt, G. (1995). Percent: A privileged proportion. *Review of Educational Research*, *65*(4), 421–481. doi:10.3102/00346543065004421

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peret, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education*, *20*(1), 8–27. doi:10.2307/749095

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. doi:10.1177/0956797612466268

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*(2), 346–362. doi:10.1037/0022-0663.93.2.346

Sackur-Grisvard, C., & Léonard, F. (1985). Intermediate cognitive organizations in the process of learning a mathematical concept: The order of positive decimal numbers. *Cognition and Instruction*, *2*(2), 157–174. doi:10.1207/s1532690xci0202_3

Schaie, K. W. (1993). The Seattle longitudinal studies of adult intelligence. *Current Directions in Psychological Science*, *2*(6), 171–175.

Sformo, T. (2008). *Practical problems in mathematics: For automotive technicians*. Independence, KY: Cengage Learning.

Siegler, R. S., Carpenter, T. P., Fennell, F., Geary, D. C., Lewis, J., Okamoto, Y., et al. (2010). *Developing effective fractions instruction for kindergarten through 8th grade: A practice guide*. Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. http://eric.ed.gov/ERICWebPortal/recordDetail?accno=ED512043

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., et al. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697. doi:10.1177/0956797612440101

Siegler, R. S., & Lortie-Forgues, H. (2015). Conceptual knowledge of fraction arithmetic. *Journal of Educational Psychology*, *107*(3), 909–918. doi:10.1037/edu0000025

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology*, *49*(10), 1994–2004. doi:10.1037/a0031200

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273–296. doi:10.1016/j.cogpsych.2011.03.001

Silver, E. A. (1983). Probing young adults' thinking about rational numbers. *Focus on learning problems in mathematics*, *5*, 105–117.

Smith, M. S., Silver, E. A., & Stein, M. K. (2005). *Improving instruction in rational numbers and proportionality* (Vol. 1). New York: Teachers College Press.

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction*, *14*(5), 503–518. doi:10.1016/j.learninstruc.2004.06.015

Steffe, L. P. (2001). A new hypothesis concerning children's fractional knowledge. *Journal of Mathematical Behavior*, *20*(3), 267–307. doi:10.1016/S0732-3123(02)00075-5

Stigler, J. W., Givvin, K. B., Thompson, B. J., Stigler, J. W., & Thompson, B. J. (2010). What community college developmental mathematics students understand about mathematics. *Mathematics Teacher*, *1*(3), 4–16. http://statlit.org/pdf/2009CarnegieFoundation-Developmental-Math-CC-Students-Understand.pdf

Sweeney, E. S., & Quinn, R. J. (2000). Concentration: Connecting fractions, decimals & percents. *Mathematics Teaching in the Middle Schooliddle school*, *5*, 324–328. http://search.proquest.com/docview/231158993?accountid=9902

Tian, J., & Siegler, R. S. (2017, April). *Influence of Number of Digits on Rational Number Magnitude Understanding*. Poster presented at the biennial meeting of the Society for Research in Child Development, Austin, TX.

Tirosh, D., Fischbein, E., Graeber, A. O., & Wilson, J. W. (1999). Prospective elementary teachers' conceptions of rational numbers, 1–17.

Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R. S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, *37*, 5–13. doi:10.1016/j.learninstruc.2014.03.002

Vamvakoussi, X., Christou, K. P., Mertens, L., & Van Dooren, W. (2011). What fills the gap between discrete and dense? Greek and Flemish students' understanding of density. *Learning and Instruction*, *21*(5), 676–685. doi:10.1016/j.learninstruc.2011.03.005

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *Journal of Mathematical Behavior*, *31*(3), 344–355. doi:10.1016/j.jmathb.2012.02.001

Vamvakoussi, X., & Vosniadou, S. (2007). How many numbers are there in a rational numbers interval ? Constraints, synthetic models and the effect of the number line. In S. Vosniadou, A. Baltas, & X. Vamvakoussi (Eds.), *Reframing the Conceptual Change Approach in Learning and Instruction* (pp. 265–282). Oxford, UK: Elsevier.

Vamvakoussi, X., & Vosniadou, S. (2010). How Many Decimals Are There Between Two Fractions? Aspects of Secondary School Students' Understanding of Rational Numbers and Their Notation. *Cognition and Instruction*, *28*(2), 181–209. doi:10.1080/07370001003676603

Varma, S., & Karl, S. R. (2013). Understanding decimal proportions: Discrete representations, Parallel access, And privileged processing of zero. *Cognitive Psychology*, *66*(3), 283–301. doi:10.1016/j.cogpsych.2013.01.002

Wang, Y. Q., & Siegler, R. S. (2013). Representations of and translation between common fractions and decimal fractions. *Chinese Science Bulletin*, *58*(36), 4630–4640. doi:10.1007/s11434-013-6035-4

Zhang, L., Wang, Q., Lin, C., Ding, C., & Zhou, X. (2013). An ERP study of the processing of common and decimal fractions: How different they are. *PLoS ONE*, *8*(7). doi:10.1371/journal.pone.0069487